Mihailo Petrović ALAS

Life Work Times

Serbian Academy of Sciences and Arts

MIHAILO PETROVIĆ 150th ALAS birth anniversary


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SERBIAN ACADEMY OF SCIENCES AND ARTS

MIHAILO PETROVIĆ ALAS: LIFE, WORK, TIMES
ON THE OCCASION OF THE 150th ANNIVERSARY OF HIS BIRTH

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LIFE, WORK, TIMES

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Exclusive editions, such as this monograph, call for the engagement, enthusiasm and cooperation of a number of individuals and institutions. We would like to use this opportunity and extend our gratitude to everyone who has taken part or in any way contributed to, or supported the creation and publication of this monograph.

First of all, we would like to express our gratitude to the authors of papers for their effort taken to provide expert and high level insights into some main points of Mihailo Petrović Alas' life and work, at the same time preserving an important aspect of being easy to read and appealing to a broader readership. In addition, we would like to thank to Ms. Snežana Krstić-Bukarica and Ms. Nevena Đurđević from SASA Publishing Section for performing a thorough proofread of the papers, thus making the writing even more articulate.

The monograph features a number of photographs and the copies of documents that have been obtained owing to the kindness of the SASA Archive, SASA Library, SASA Mathematical Institute, Archive of Serbia, Mr. Viktor Lazić from the "Adligat" Society, Mr. Jovan Hans Ivanović and his "Mihailo Petrović Alas" Foundation, "Mihailo Petrović Alas" Primary School, "Svetozar Marković" University Library, Belgrade City Museum, Zavod za udžbenike (Institute for Textbook Publishing) in Belgrade, Virtual Library of Faculty of Mathematics in Belgrade and Digital Legacy of Mihailo Petrović Alas.

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S. Pilipović, G. Milovanović, Ž. Mijajlović

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EDITOR'S FOREWORD

As soon as one first encounters the work of Mihailo Petrović, it becomes evident that he was a person that according to its numerous traits was a polymath. Above all, the academician Petrović was a gifted mathematician and a renowned professor at the University of Belgrade, but also a fisherman, writer, philosopher, musician, world traveler and a travel writer. He earned a degree in mathematics at the Belgrade Grand School and a licentiate degree in mathematics, physics and chemistry at the Sorbonne. At the age of 26, only a year after he had completed his studies, he defended his PhD degree in mathematics at the same university, as a student of the famous French mathematicians Henri Poincaré, Charles Hermite and Charles Émile Picard. In the same year (1894) he was elected to the position of professor at the Grand School to which he brought the spirit of the French mathematical school. It was at that point that his long and prolific journey through science began, whereas, owing to him, Belgrade achieved parity with other major European centers in mathematical sciences. He became an initiator and a leader of the Serbian mathematics and strongly contributed to the spirit of the modern European science in Serbia.

Petrović's expertize spanned several mathematical areas in which he achieved scientific results of world-class relevance: differential equations, numerical analysis, theory of functions of a complex variable and geometry of polynomials. He was also interested in natural sciences, chemistry, physics and biology, and he published scientific papers in these fields, too. In his scientific endeavor he managed to meet the most rigorous standards of the most developed European countries. In a brilliant rise, in a few years' time, up to the early 20th century, he wrote around thirty papers that he published in the leading European mathematical journals. It was due to this fact that he was elected a member of the Serbian Royal Academy as early as at the age of 30, and soon after he became a member of a number of foreign academies and prominent expert societies. He won the greatest respect of the global mathematical community: he was among few mathematicians (13) who delivered at least five plenary lectures or lectures as a visiting lecturer at the International Congress of Mathematicians (ICM). He delivered five such lectures (1908, 1912, 1924, 1928 and 1932). One such invitation has been considered by the mathematical community as an equivalent of an induction to a hall of fame. In addition, it has been considered that Petrović was a founder of new scientific disciplines, namely mathematical phenomenology and spectral theory. He invented several analogue computing machines, possessed technical patents and was the main cryptographer of the Serbian and Yugoslav Army.

Up to the Second World War he was the mentor of all doctoral thesis in mathematics defended at the University of Belgrade. Aforementioned is related to one of professor Petrović's greatest and most important achievements – he was a founder of the Serbian mathematical school that has produced a great number of renowned and successful mathematicians not only in Serbia but also around the world.

In 2018, the Serbian Academy of Sciences and Arts and mathematicians in Serbia celebrate the 150th anniversary of the birth of Mihailo Petrović Alas. Throughout this year, the Academy has organized a large exhibition dedicated to Petrović, alongside a solemn gathering and a conference. This monograph commemorates this important jubilee of the Serbian mathematics. Given the fact that a lot of articles on Petrović have already been written, and that his collected works were published at the end of the last century, the editors and authors of the papers in this monograph were faced with a daunting task of finding some new details from professor Petrović's life and career. Even more so given that his body of work is immense, spanning different scientific areas and encompassing topics that at first glance one finds difficult to combine. As Dragan Trifunović, Petrović's biographer and a man who most thoroughly studied his life and work, noted on one occasion that almost an institute was necessary that would encompass professor's entire body of work. Therefore, we set a relatively modest goal to ourselves to shed light upon some main points of Petrović's life and work, times and circumstances he lived in, as well as to elaborate on the present developments in relation to the Serbian mathematical school, through a selection of papers. The authors of the papers steered clear of technical details and excessive use of mathematical language. Hence, the monograph is intended for a broader readership, in particular to those readers who are interested in the history of Serbian science and its evolvement at the turn of the 20th century, but also to those who want to gain a deeper insight into the life of a brilliant mathematician and a polymath, and, we can quite freely say, an unusual personality.

Ž. Mijajlović, S. Pilipović, G. Milovanović

MIHAILO PETROVIĆ ALAS: LIFE AND WORK

MIHAILO PETROVIĆ ALAS – SCIENTIFIC LEGACY AND MODERN ACHIEVEMENTS IN PROBABILITY THEORY

Dora SELEŠI
University of Novi Sad,
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"Problems, nowadays intractable by the tools of mathematical analysis, do not have to stay that way for all eternity; difficulties, invincible for today's mathematics, may become no more than a piece of cake for tomorrow's one."

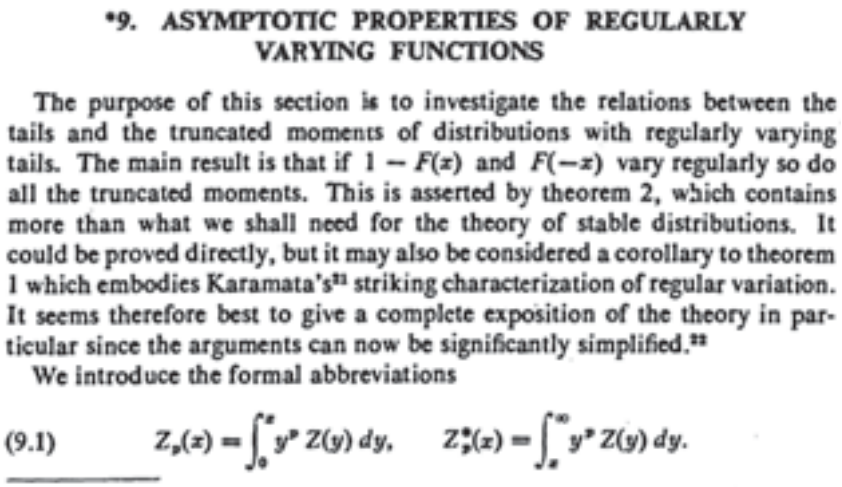
Mihailo Petrović, 1914.

The Danube and the Sava – two rivers, two shores where the prominent Alas grew up and reached adulthood, found his inspiration, purpose and home. Calm, dignified and humble at their source, they turn into mighty giants. Their continuous flows carry great riches. Differential equations and special functions – the two fields that were the main focus of Mihailo Petrović's research where he made substantial contributions. The results, which may seem simple and modest in terms of today's evaluation criteria, are continuously evolving and turning into mighty achievements, embodying a source of great scientific knowledge. Time flows and carries centuries in its stream, the generations of mathematicians are maturing and working all over the world. The scientific legacy of professor Petrović remains as relevant today as ever. In this article I will try to briefly present the current state of the art of mathematical

A photo of the Danube in Zemun, an old picture postcard (source: https://www.kolekcionar.net/articles.php?id=278795)

analysis, with an emphasis on stochastic analysis and some modern scientific achievements resulting from Mihailo Petrović's papers.

Mihailo Petrović (1868–1943), a doyen of mathematics in Serbia, took his first steps in scientific research with his papers elaborating on the theory of ordinary differential equations and he left a vast amount of papers in this field. Soon he discovered his other field of permanent interest: the theory of special functions, mainly elliptic functions. He made a significant contribution to the theory of differential equations and their solving via power series, as well as to the approximation of functions and the qualitative analysis of differential equations. The value of these works exceeds the time in which they were written; the modelling of physical phenomena nowadays relies not only on a deterministic approach via ordinary differential equations, but also on quantifying the uncertain factors by probabilities and incorporating them into the equation as stochastic (random) processes. Hence, many natural, social, economic and other phenomena can be appropriately described via *stochastic differential equations*. One of the most advanced methods for solving stochastic differential equations is the method of polynomial chaos expansions, whose essence lies in expressing a stochastic process as a Fourier series via the base of stochastic orthogonal polynomials (Hermite, Jacobi, Legendre and other polynomials). In this idea one can



*9. ASYMPTOTIC PROPERTIES OF REGULARLY VARYING FUNCTIONS

The purpose of this section is to investigate the relations between the tails and the truncated moments of distributions with regularly varying tails. The main result is that if $1 - F(x)$ and $F(-x)$ vary regularly so do all the truncated moments. This is asserted by theorem 2, which contains more than what we shall need for the theory of stable distributions. It could be proved directly, but it may also be considered a corollary to theorem 1 which embodies Karamata's[21] striking characterization of regular variation. It seems therefore best to give a complete exposition of the theory in particular since the arguments can now be significantly simplified.[22]

We introduce the formal abbreviations

$$(9.1) \qquad Z_\rho(x) = \int_0^x y^\rho Z(y)\,dy, \qquad Z_\rho^*(x) = \int_x^\infty y^\rho Z(y)\,dy.$$

* This section is used only for the theory of stable distributions, but the use of theorem 2 would simplify many lengthy calculations in the literature.

[20] Special cases were noticed by S. Port.

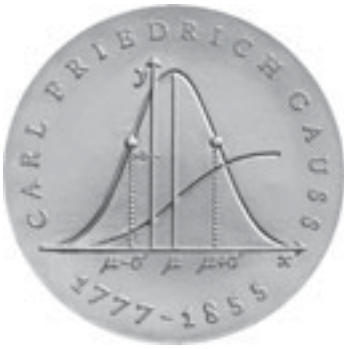
[21] J. Karamata, *Sur un mode de croissance regulière*, Mathematica (Cluj), vol. 4 (1930) pp. 38–53. Despite frequent references to this paper, no newer exposition seems to exist. For recent generalizations and applications to Tauberian theorems see W. Feller, *One-sided analogues of Karamata's regular variation*, in the Karamata memorial volume (1968) of L'Enseignement Mathématique.

[22] Although new, our proof of theorem 1 uses Karamata's ideas.

William Feller was the first to apply Karamata's theory of regularly varying functions in probability theory. The image depicts a page from Feller's book.

perceive not only a modern solving technique that originated from the theory of approximations and special functions, but also the essence of Petrović's phenomenology in the very setup of the model. Henri Poincare (1854–1912) claimed that differential equations are inherently involved in all laws of nature and science. Petrović adopted this idea and used it to develop his theory that he termed mathematical phenomenology. A contemporary designation of the same concept is *mathematical modeling*. The modeling of stochastic nonlinear dynamical systems is the main focal point of research related to modern mechanics, astronomy, sociology, biology, economy and many other fields of science. Problems related to stochastic differential equations that describe the models of fluid flow, blood flow in the vessels, optimal aircraft design, spreading of epidemics, price changes of financial derivatives, etc., are just some of the most interesting examples of equations the solving of which occupies today's mathematicians.

Jovan Karamata (1903–1967), who was Mihailo Petrović's student and the creator of the theory of regularly varying functions and slowly varying functions, was the one who made the most significant contribution to probability theory. The importance of this class of functions came to the center of attention in Feller's book titled *An Introduction to Probability*



A coin of the Deutsche Mark 20 DEM, produced in 1977, on the occasion of Gauss's birthday, depicting Gauss's curve of normal distribution.

Theory and its Applications (*W. Feller, An Introduction to Probability Theory and its Applications I and II, Wiley, New York, 1969 and 1971*). The famous central limit theorem in probability theory states that sums of independent random variables with finite variances behave as the normal (Gaussian) distribution in limit process. It is only in the last few years that random variables with infinite variances were also proven to be realistic models for various phenomena, e.g. for insurance claims arising from natural disasters (earthquakes, floods, hurricanes, etc.), for magnitude levels of ground shaking during earthquakes, for file sizes sent via e-mail, for the time spent on social networks, etc. The central limit theorem does not necessarily hold for variables with infinite variances, thus in order to determine the distribution of sums and of maxima/minima of such variables, the classes of Karamata's distribution functions with slowly varying tails must be used. A modern theory covering this topic is *the theory of large deviations* and it is being applied in insurance theory, risk management, information theory, queueing theory, renewal theory, thermodynamics and statistical mechanics.

One branch of mathematicians in Serbia who deal with probability theory and stochastic analysis has grown out of Karamata's node in a genealogical tree of Mihailo Petrović. Karamata's student Bogoljub Stanković (1924–2018), followed by his student Stevan Pilipović, has brought the research of functional analysis and generalized functions to Serbia, whereas the study of generalized stochastic processes has been formed as a special sub-field of this research. Within Stanković's Novi Sad school of mathematics at the University of Novi Sad, contributions to probability theory and generalized stochastic processes were made by Olga Hadžić, Stevan Pilipović, Mila Stojaković, Zagorka Lozanov Crvenković, Danijela Rajter Ćirić and the author of this article. The other branch of probabilisticians in Serbia and Petrović's successors has grown out of the node of Tadija Pejović (1892–1982) and his PhD candidate Zoran Ivković (1934–2011). The vast majority of the professors at the Universities in Belgrade and Niš who work in the field of probability theory and statistics are Ivković's successors: Jovan Mališić and Svetlana Janković from the first generation, then Pavle Mladenović, Biljana Popović and Miljana Jovanović as the second generation of mathematicians, and many of their PhD candidates and mathematical descendants of the third and fourth generation: Miroslav Ristić, Marija Milošević as well as many others. A detailed outline of the genealogical tree can be seen on the *Mathematics Genealogy Project* portal. Of course, we

must mention the names of those in our school of probability who do not stem from Petrović, such as Milan Merkle who obtained his PhD degree in Michigan or Slobodanka Janković who, via Stevan Stojanović, stems from Đuro Kurepa (1907–1993), who was of great influence in our region. It is possible that many names are missing in this retrospective, but, as this work is rather a personal impression, and not a document which claims to be precise in all details, the author apologizes to all those whose names are unjustifiably omitted.

Mihailo Petrović was a universal creator and thinker, a true polymath, scientist, mathematician, inventor, philosopher, ichthyologist, musician, writer, world traveler. Many authors regard him, along with Norbert Wiener (1894–1964) and Johann von Neumann (1903–1957), as a forefather of cybernetics and computer science. The far-reaching influence of his works reaches even today's most modern fields of science such as neural networks, artificial intelligence, machine learning, deep learning, which are all inspired by the cognitive functioning of the human brain trying to imitate its activity by using mathematical algorithms. Google's algorithm for searching and indexing of web pages, the formalization of natural language learning and the creation of a universal translator are just a few examples of a more or less successful implementation of these techniques. Integral transformations, function approximations and power series are the roots of what is today used in signal processing, digital image processing, with applications in mobile communications, biomedicine, computerized tomography and many other areas of science, engineering and economy. An integral part of all these models are noises that occur in transformation channels, errors in instrument readings, etc., which are quantified as uncertainties i.e. as random processes, and they introduce probability, statistics and stochastic analysis into the contemporary models.

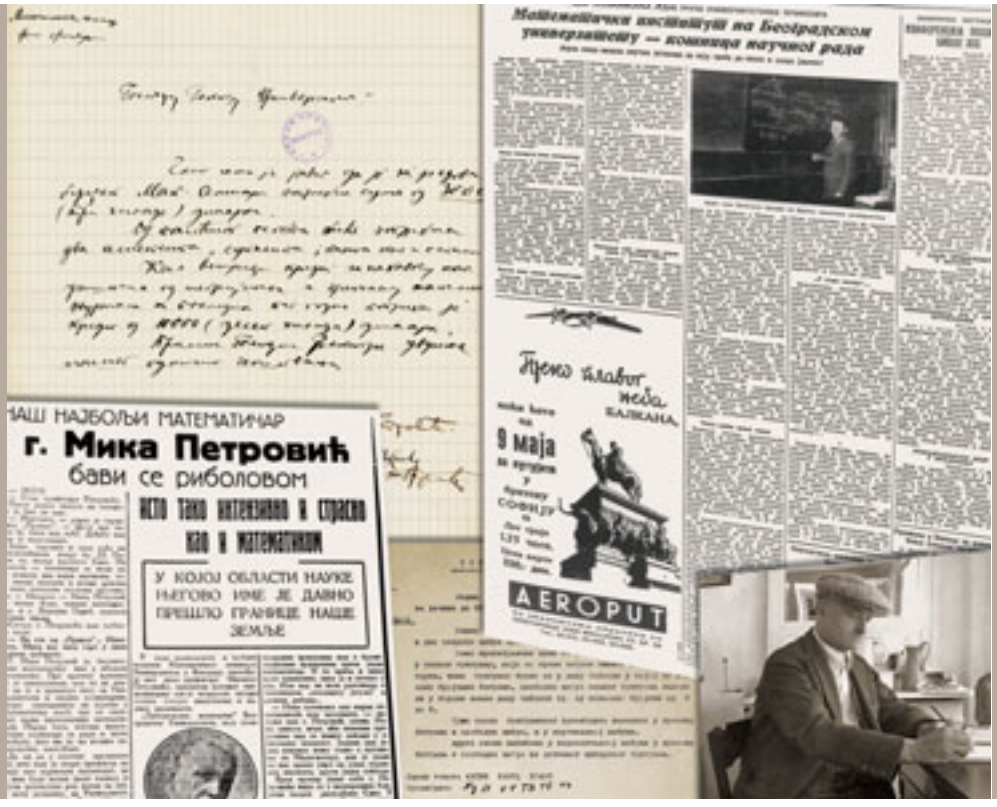
The man who spent his life at the confluence of the Sava and the Danube was a mathematician who was equally engaged with analysis and algorithms and who understood the intrinsic ties among continuity, continuum and discrete structures. Regardless whether he was inspired by the calm, continuous flows of rivers to embark on the analysis of differential equations and continuous structures, or the smooth water streams to notice direction fields and integral curves within them, or his favorite leisure activity – fishing – inspired him to adopt a more algorithmic way of thinking, it will forever remain a mystery. The fact is

Jovan Karamata postmark, published in 2002

Mihailo Petrović Alas postmark, published in 1993

that there is a huge body of scientific papers he left behind, in which continuous objects are approximated by discrete ones (let us remember, for instance, the Riemann integral or the expansion of a function into a power series, the numerical solving of equations, etc.) and discrete objects are approximated by continuous (for instance, in analytical number theory). The large gap between various areas of mathematics needs to be bridged, as much more can be achieved by accepting the natural unity of mathematics and the unity in its duality. Algebraic topology is a brilliant example which underlines the needs and benefits of the synthesis of a broader spectrum of mathematical disciplines. The explosive development of probability theory in the last few years and the fact that it permeates all spheres of mathematics might play the role of this connecting bridge. Probabilistic logic, random graphs and algorithms of random decision trees, Monte Carlo methods in modeling, distributed optimization, quantum computers are just a few examples in which probability theory meets classical discrete mathematics or computer science. On the specific level of stochastic analysis, positive examples that point to a synthesis of various areas are Markov processes which are connected to semigroup theory, singular stochastic processes that are defined via Colombeau's algebras of generalized functions, regularity structures that are used for solving stochastic partial differential equations by regularization methods and quantum field theory methods, and it is exactly these examples that are determining the further direction for development of modern stochastic analysis as an integral part of the scientific legacy of Mihailo Petrović Alas.

Mihailo Petrović's memory from the Danube (SASA Archive, 14188/12)


НАШ НАЈБОЉИ МАТЕМАТИЧАР
г. Мика Петровић
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ИСТО ТАКО ИНТЕРЕСНО И СТРАСНО КАО И МАТЕМАТИКОМ
У КОЈОЈ ОБЛАСТИ НАУКЕ НЕГОВО ИМЕ ЈЕ ДАВНО ПРЕШЛО ГРАНИЦЕ НАШЕ ЗЕМЉЕ
Математички институт на Београдском универзитету — комплекс научног рада
Преко плавог неба Балкана
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AEROPUT

MIHAILO PETROVIĆ
IN THE MEDIA AND ACHIVES