Mihailo Petrović ALAS

Life Work Times

Serbian Academy of Sciences and Arts

MIHAILO PETROVIĆ ALAS
150th birth anniversary

MIHAILO
PETROVIĆ
150th ALAS
birth anniversary

SERBIAN ACADEMY OF SCIENCES AND ARTS

MIHAILO PETROVIĆ ALAS: LIFE, WORK, TIMES
ON THE OCCASION OF THE 150th ANNIVERSARY OF HIS BIRTH

Publisher
Serbian Academy of Sciences and Arts
Knez Mihailova 35, Belgrade

Acting publisher
Academician Vladimir S. Kostić

Editor-in-chief
Academician Marko Anđelković

Editors of publication
Academician Stevan Pilipović
Academician Gradimir V. Milovanović
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Cover design
Dragana Lacmanović-Lekić

Prepress
Dosije Studio, Belgrade

Selection of artworks
Maja Novaković

English translation
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Tanja Ružin Ivanović, Žarko Radovanov, Dora Seleši

Proofreading and editing
Jelena Mitrić

Printing
Planeta print, Belgrade

Print run: 500 copies

ISBN 978-86-7025-818-1

The publication was financially supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia and Telekom Srbija.

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LIFE, WORK, TIMES

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OF HIS BIRTH

SERBIAN ACADEMY OF SCIENCES AND ARTS

Exclusive editions, such as this monograph, call for the engagement, enthusiasm and cooperation of a number of individuals and institutions. We would like to use this opportunity and extend our gratitude to everyone who has taken part or in any way contributed to, or supported the creation and publication of this monograph.

First of all, we would like to express our gratitude to the authors of papers for their effort taken to provide expert and high level insights into some main points of Mihailo Petrović Alas' life and work, at the same time preserving an important aspect of being easy to read and appealing to a broader readership. In addition, we would like to thank to Ms. Snežana Krstić-Bukarica and Ms. Nevena Đurđević from SASA Publishing Section for performing a thorough proofread of the papers, thus making the writing even more articulate.

The monograph features a number of photographs and the copies of documents that have been obtained owing to the kindness of the SASA Archive, SASA Library, SASA Mathematical Institute, Archive of Serbia, Mr. Viktor Lazić from the "Adligat" Society, Mr. Jovan Hans Ivanović and his "Mihailo Petrović Alas" Foundation, "Mihailo Petrović Alas" Primary School, "Svetozar Marković" University Library, Belgrade City Museum, Zavod za udžbenike (Institute for Textbook Publishing) in Belgrade, Virtual Library of Faculty of Mathematics in Belgrade and Digital Legacy of Mihailo Petrović Alas.

The publication of the monograph was financially supported by JP Srbijagas, the Ministry of Education, Science and Technological Development, primarily through scientific projects in which the majority of the authors of the papers takes part, and Telekom Srbija. We would like to express our deep gratitude for their support.

Finally, we would like to express our gratitude to Mr. Mirko Milićević from the publishing house "Dosije Studio" for excellent prepress preparation of the monograph.

S. Pilipović, G. Milovanović, Ž. Mijajlović

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EDITOR'S FOREWORD

As soon as one first encounters the work of Mihailo Petrović, it becomes evident that he was a person that according to its numerous traits was a polymath. Above all, the academician Petrović was a gifted mathematician and a renowned professor at the University of Belgrade, but also a fisherman, writer, philosopher, musician, world traveler and a travel writer. He earned a degree in mathematics at the Belgrade Grand School and a licentiate degree in mathematics, physics and chemistry at the Sorbonne. At the age of 26, only a year after he had completed his studies, he defended his PhD degree in mathematics at the same university, as a student of the famous French mathematicians Henri Poincaré, Charles Hermite and Charles Émile Picard. In the same year (1894) he was elected to the position of professor at the Grand School to which he brought the spirit of the French mathematical school. It was at that point that his long and prolific journey through science began, whereas, owing to him, Belgrade achieved parity with other major European centers in mathematical sciences. He became an initiator and a leader of the Serbian mathematics and strongly contributed to the spirit of the modern European science in Serbia.

Petrović's expertize spanned several mathematical areas in which he achieved scientific results of world-class relevance: differential equations, numerical analysis, theory of functions of a complex variable and geometry of polynomials. He was also interested in natural sciences, chemistry, physics and biology, and he published scientific papers in these fields, too. In his scientific endeavor he managed to meet the most rigorous standards of the most developed European countries. In a brilliant rise, in a few years' time, up to the early 20th century, he wrote around thirty papers that he published in the leading European mathematical journals. It was due to this fact that he was elected a member of the Serbian Royal Academy as early as at the age of 30, and soon after he became a member of a number of foreign academies and prominent expert societies. He won the greatest respect of the global mathematical community: he was among few mathematicians (13) who delivered at least five plenary lectures or lectures as a visiting lecturer at the International Congress of Mathematicians (ICM). He delivered five such lectures (1908, 1912, 1924, 1928 and 1932). One such invitation has been considered by the mathematical community as an equivalent of an induction to a hall of fame. In addition, it has been considered that Petrović was a founder of new scientific disciplines, namely mathematical phenomenology and spectral theory. He invented several analogue computing machines, possessed technical patents and was the main cryptographer of the Serbian and Yugoslav Army.

Up to the Second World War he was the mentor of all doctoral thesis in mathematics defended at the University of Belgrade. Aforementioned is related to one of professor Petrović's greatest and most important achievements – he was a founder of the Serbian mathematical school that has produced a great number of renowned and successful mathematicians not only in Serbia but also around the world.

In 2018, the Serbian Academy of Sciences and Arts and mathematicians in Serbia celebrate the 150th anniversary of the birth of Mihailo Petrović Alas. Throughout this year, the Academy has organized a large exhibition dedicated to Petrović, alongside a solemn gathering and a conference. This monograph commemorates this important jubilee of the Serbian mathematics. Given the fact that a lot of articles on Petrović have already been written, and that his collected works were published at the end of the last century, the editors and authors of the papers in this monograph were faced with a daunting task of finding some new details from professor Petrović's life and career. Even more so given that his body of work is immense, spanning different scientific areas and encompassing topics that at first glance one finds difficult to combine. As Dragan Trifunović, Petrović's biographer and a man who most thoroughly studied his life and work, noted on one occasion that almost an institute was necessary that would encompass professor's entire body of work. Therefore, we set a relatively modest goal to ourselves to shed light upon some main points of Petrović's life and work, times and circumstances he lived in, as well as to elaborate on the present developments in relation to the Serbian mathematical school, through a selection of papers. The authors of the papers steered clear of technical details and excessive use of mathematical language. Hence, the monograph is intended for a broader readership, in particular to those readers who are interested in the history of Serbian science and its evolvement at the turn of the 20th century, but also to those who want to gain a deeper insight into the life of a brilliant mathematician and a polymath, and, we can quite freely say, an unusual personality.

Ž. Mijajlović, S. Pilipović, G. Milovanović

MIHAILO PETROVIĆ ALAS: LIFE AND WORK

THE HYDROINTEGRATOR OF MIHAILO PETROVIĆ ALAS

Radomir S. STANKOVIĆ
Mathematical Institute of SASA

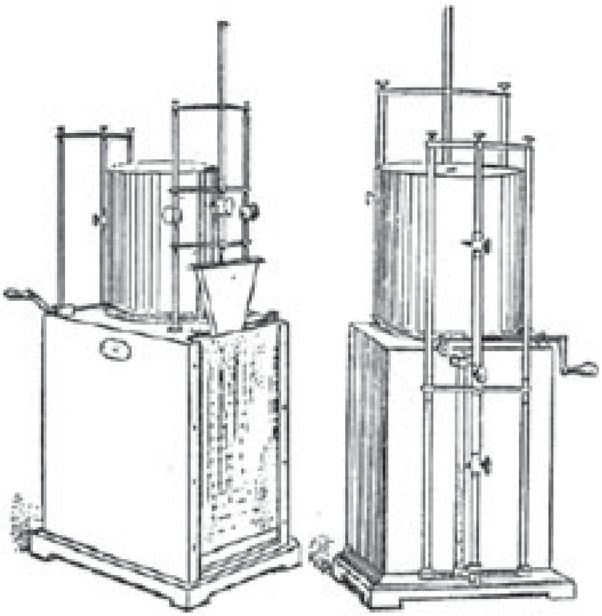
Building computing machines and other technical devices for the acceleration and automatization of calculating procedures has been a subject of scientific interest for centuries, and even millenniums, to which, for instance, *Antikythera mechanism*[169] bears testimony.

Taking into account the writings of Mihailo Petrović Alas on the equipment used for fishing and the detailed guidelines for its usage, it is hardly surprising that this eclectic research scientist turned his practical character towards building devices for solving specific mathematical problems. Having in mind the topic of his doctoral dissertation (Petrović 1894), it is completely understandable that he had chosen differential equations as the subject of his research within that field.

As his nickname – Alas ("a river fisherman"), clearly indicates, Mihailo Petrović spent a lot of time fishing on the Danube and Sava rivers, and it is only natural that he had chosen water as the main element of his device for solving differential equations, which he simply dubbed the "hydrointegrator".

In this paper we are going to describe the basic parts of the hydrointegrator, as well as its working principles, give comments on one specific realization, as well as on the potential improvements suggested by Petrović, and with a view to all that was previously said, name the contributions of Mihailo Petrović Alas to building analogue computers.

HYDROINTEGRATOR



Petrović's sketch of the hydrointegrator
(Digital Legacy of Mihailo Petrović)

The originality and effectiveness of a solution is probably more important than emphasizing the primacy of an invention, but it should still be noted that the hydrointegrator of Mihailo Petrović Alas is the world's first analogue computing machine for integrating differential equations based on hidraylic principles. Up until then the devices based on the principle of the law of hydraulics had been suggested only for solving algebraic equations, such as Veltmann's machine of 1889.

The working principle of the hydrointegrator is based on noticing the equivalence of laws of changes to water levels when a body is immersed into it and the procedure for integrating certain types of differential equations, including Riccati's equation[170], in whose solving Petrović was particularly interested. With a view to that, the hydrointegrator can be considered to be what Petrović calls the materialization of mathematics as the ultimate goal of mathematical phenomenology, seen as drawing analogies between certain mathematical problems and physical, often disparate, phenomena, with a view to resolving the former, as noted in Petrović's publications on the said topic (Петровић 1911; Petrović 1921).

The hydrointegrator represents the result of Petrović's considerations and research of many years, to which testify his notes from the lectures on mechanics given by professor König at *Collège de France* in Paris, which Petrović attended in 1892. These notes contain elements of the input–output system that Petrović called integraph. It represents the devices for calculating the surface of irregular geometrical figures circumscribed by closed contours. The corresponding page from the notes of Mihailo Petrović is reproduced in Trifunović (see Трифуновић 1968a: 459, Picture 112).

Petrović later applied these considerations and other original solutions to a practical realization of the hydrointegrator. Let us mention that Petrović's work on that problem had been announced as early as 1896 by his senior colleague Ljubomir Klerić, a professor of mechanics at the Faculty of Philosophy of Belgrade Grand School, placing a special emphasis on the originality of his approach.

PETROVIĆ’S PUBLICATIONS ON THE SUBJECT OF THE HYDROINTEGRATOR

In accordance with his principles of properly documenting proposed solutions, Petrović published four papers containing a detailed description of the working principle, building process and practical applications of the hydrointegrator. It was first presented at the French Academy of Sciences, by professor Paul Émile Appell, on 17 May, 1897. It is important to note that, due to its significance, this paper was reprinted in *Journal de Physique* (Petrović 1897: 476–479), departing from its usual practice.

Petrović published this paper in Serbian, with certain additions, in 1898, in *Srpski tehnički list (Serbian Technical Journal)*, a bulletin of the Association of Serbian Engineers and Architects, as noted in the subtitle of this magazine (Петровић 1898). The front page of this magazine is shown in Figure 1. The following two publications dedicated to the hydrointegrator were published by Petrović in French in the *American Journal of Mathematics,* in 1898 and 1899, respectively (Petrović 1899a; 1899b).

СРПСКИ

ТЕХНИЧКИ ЛИСТ

ОРГАН УДРУЖЕЊА СРПСКИХ ИНЖЕЊЕРА И АРХИТЕКТА

УПРАВНИ ОДБОР УДРУЖЕЊА

ГОДИНА IX.	ЈАНУАР и ФЕБРУАР 1898.	СВЕСКА 1. и 2.

ИЗ НАУКЕ И ПРАКСЕ

О хидрауличкој интеграцији

Figure 1. The cover page of *The Serbian Technical Journal* containing the paper *On hydraulic integration* by Mihailo Petrović Alas and a schematic diagram of the hydrointegrator's working principle

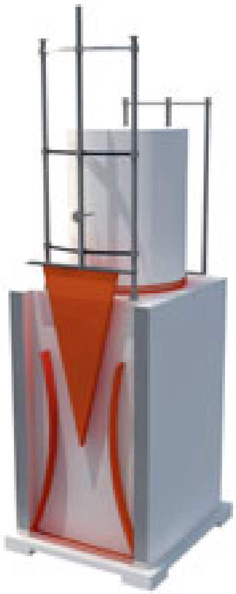


Figure 2 (left): 3D model of the hydrointegrator, author of 3D model of the hydrointegrator Petar Vranić (Mathematical Institute of SASA)

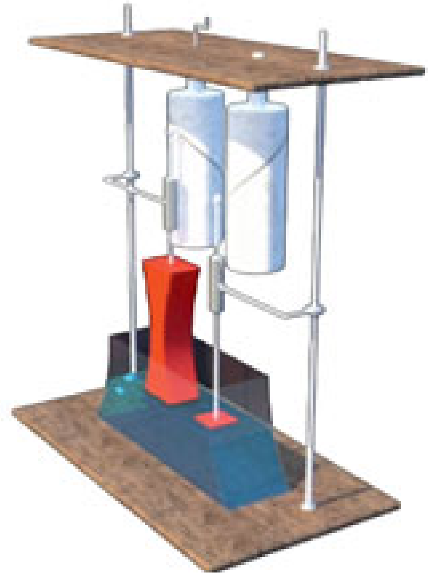


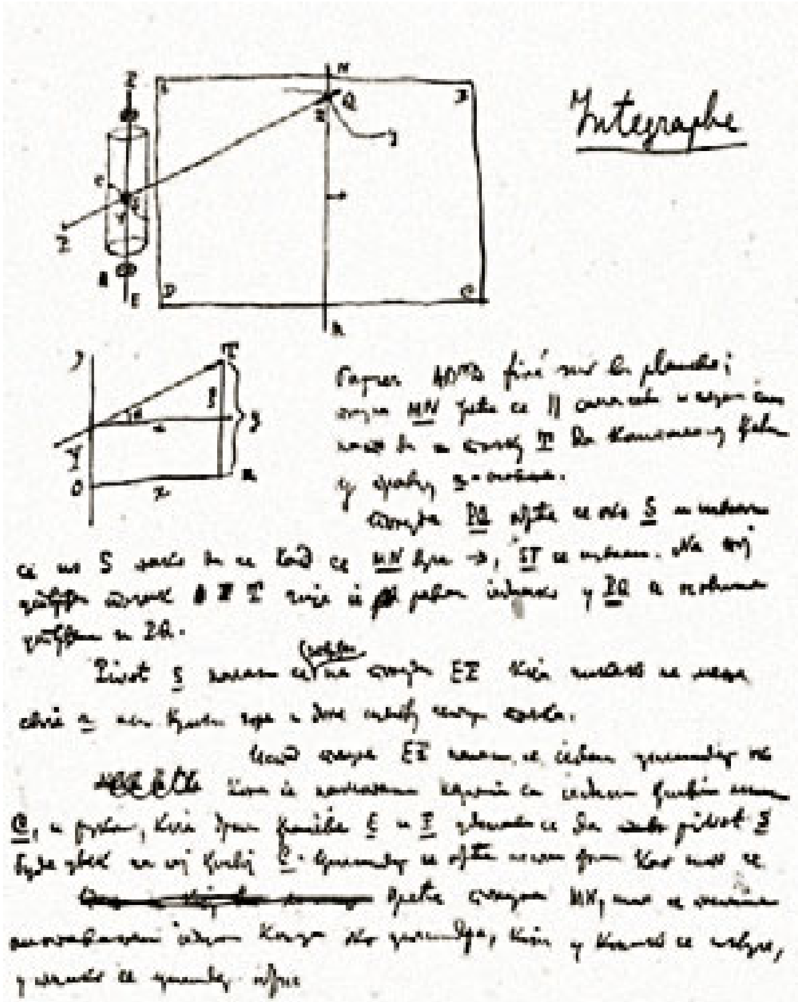
Figure 3. 3D model of basic elements of a hydrointegrator with a rotating input roller, author of 3D model Petar Vranić (Mathematical Institute of SASA)

ELEMENTS OF THE HYDROINTEGRATOR'S CONSTRUCTION

The hydrointegrator, viewed as a computing device, consists of the following functional units:

1. *Input unit*, which is in essence a body of a certain shape with or without a rotating cylinder. The function of the body shape represents the input data for the hydrointegrator. In the case of there being added a rotating cylinder, the additional input data represents the function determining the way in which the bodies are immersed in a water-filled tank. This function is realized by a groove in the rotating cylinder which corresponds to the graph of the selected function. The shaft of the body being immersed in the water slides along the groove.
2. *Arithmetical unit*, which comprises a tank of a certain shape filled with water, a body of the precisely defined shape, belonging to the input unit, which is immersed in the tank, and a floating buoy, which is used for determining the water level in the tank.
3. *Output unit*, which is in the shape of a rotating cylinder, with a pencil for tracing solutions on the paper fastened onto the surface of the cylinder.

As previously mentioned, using the rotating cylinder for tracing solutions, which can be said to represent an original contribution to building analogue computers, had been considered by Petrović as early as during his stay in Paris in 1892. He presented this idea as a technical solution in the first paper devoted to the hydrointegrator of 1897 (Petrović 1897).



Sketch of a rotating cylinder as an element of the hydrointegrator (Library of SASA, A40/120)

In the article published in *Srpski tehnički list* (Петровић 1898), as well as in the following two papers on this subject (Petrović 1899a, 1899b), Mihailo Petrović examines several versions of hydrointegrators and proposes certain improvements.

Figure 2 shows a 3D model of the basic version of the hydrointegrator, and Figure 3 an improved version of the device with a rotating input cylinder. The improved version has never been practically realized.

Apart from adding the rotating cylinder as an element of the input unit, in (Петровић 1898) Petrović proposes that at the bottom of the tank in which the body is immersed an orifice be made through which water would continually flow out. The diameter of the orifice would be defined so as to ensure that the contraction of the current of water flowing out be complete. Thereby an additional parameter in defining the functionality of the hydrointegrator would be set up, using the quantity of the displaced water in observing changes to the water level in the tank, as a result of immersing in it a body of a certain shape from the input unit. This would enable integrating different types of differential equations, depending on defining the said parameters related to the input and arithmetical unit of the hydrointegrator.

In (Петровић 1899б) Petrović proposes the application of a clock mechanism for providing constant angular velocity and synchronization of the turning of the input and output rotating cylinders joined together by a kinematic connection. The clock mechanism would ensure that the floating buoy on the input cylinder, which is used to determine the manner of immersing the body into the water tank, always crosses the same distance in a set period of time, which improves the precision of the final result.

Owing to the kinematic connection between the input and output rotating cylinders, the hydrointegrator of Mihailo Petrović can be considered as a combination of hydraulic and kinematic analogue computing machine that is used for integrating one specific differential equation, based on the selected parameters. Changing the parameters allows for integrating a broader class of differential equations.

Bearing in mind some aspects of a practical nature, more specifically, the simplicity of replacing the tank and the immersed body, Petrović proposes the construction of a hydrointegrator in which the tank and the shaft of the body are placed sideways and turned facing outwards, so that they could easily be replaced by a tank and body of another appropriately chosen shape. Thereby the hydrointegrator can easily be reprogrammed for integrating another type of differential equations. In that sense, the pair (*body*, *tank*) can be taken to be a subprogramme in terms of modern computing terminology. The parameters used for making a selection of an equation that the hydrointegrator is to solve are the following:

1. The shape of an immersed body;
2. The shape of a tank;
3. The existence of an orifice of a pre-selected and appropriate diameter at the bottom of the tank;
4. The manner of immersing the body in the tank, that is, the choice of function according to whose graph the shaft of the immersed body is moving; and
5. The diameter of the rotating cylinders in the input and output unit.

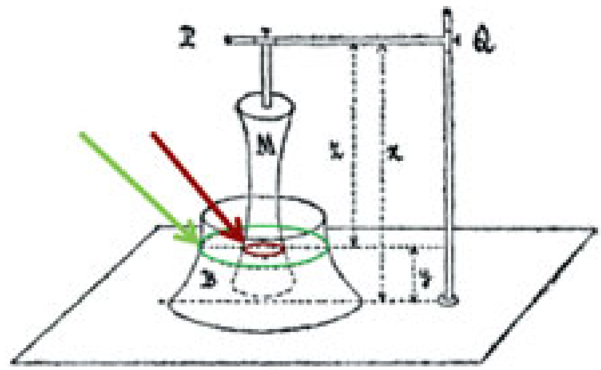


Figure 4. An illustration of hydrointegrator's working principle from the first page of *The Serbian Technical Journal*, January and February 1898, Notebook 1 and 2, figure 1.

WORKING PRINCIPLES OF THE HYDROINTEGRATOR

The basis for building the hydrointegrator represents the observation within the field of mathematical phenomenology that by an adequate choice of the shape of the body M and the tank B in which the body is immersed, they can be so adjusted that the law of changes to the water level in a tank is correlated with the procedure for integrating a previously selected differential equation.

Let us label the surfaces of the horizontal cross section of the tank B and body M as $\phi(y)$ and $F(z)$ on a plane at a distance x from the bottom of the tank, see Figure 4. Due to the immersing of the body M into the tank B, the distance of the body from the bottom of the tank changes from x to $x - dx$, while the level of fluid is raised from y to $y+dy$, so that the volume of the fluid that shifted from y to $y+dy$ equals $(\phi(y)-F(z))dy$. The volume of the displaced fluid is equivalent to the volume of the fluid taken up by the body M submerged at the value of dz, that is, $F(z)dz$, which yields

$$(\phi(y)-F(z))dy = F(z)dz.$$

As the relation $z = x - y$, is satisfied, it results in the following

$$\frac{dy}{dx} = \frac{F(x-y)}{\phi(y)} . \qquad (1)$$

The aforementioned improvement of the construction presented in (Petrović 1897) requires adding an orifice at the bottom of the tank, through which it is continually drained, and thus in that case the change to the water level in the tank is equivalent to the volume of the displaced water upon submerging the body M and the volume of water flowing out of the orifice at the bottom of the tank during the period of time dt.

As a man of practical bent, Petrović, taking into account the materials and the technology at his disposal at the time, proposed that the immersed body and tank have two flat sides parallel to each other, as well as a flat bottom part, whereas the other two sides should be arbitrarily but adequately selected curved surfaces as in Figure 5, where the breath of the tank $\phi(y)$ and the body $\theta(z)$ at the height y and z functions are freely chosen, as are the distance α and β of the parallel sides of the tank B and the body M. In that case $\Phi(y)=\alpha\phi(y)$ and $F(z) = \beta\theta(z)$, so that the equation (1) represents

$$\alpha\phi(y)\frac{dy}{dx} = \beta\Theta(x-y).$$

If the shapes of the body and tank are selected in that order, as in

$$\phi(y) = \frac{1}{\alpha f(y)} \quad \text{and} \quad \theta(z) = \frac{1}{\beta}\psi(z),$$

the hydrointegrator integrates the equation of the following form

$$\frac{dy}{dx} = f(y)\psi(x-y).$$

In the improved model of the hydrointegrator proposed by Petrović, the manner of immersing the body M would be determined by a grooved rotating input cylinder, as shown in Figure 6. Provided that the diameters of the input and output cylinders D and E are mutually different, a more general equation follows

$$\frac{dy}{dx} = f(y)\psi(ax-y).$$

If the body M is in the shape of a prism, then it holds that $\theta(z) = const. = \beta'$, so that the hydrointegrator works as an integraph for calculating the surface of the cross section of the tank in the plane of a picture

$$x = \frac{\alpha}{\beta\beta'}\int \phi(y)dy.$$

As shown in (Петровић 1898), the hydrointegrator works as an integraph even in the case when the body B is in the shape of a prysm.

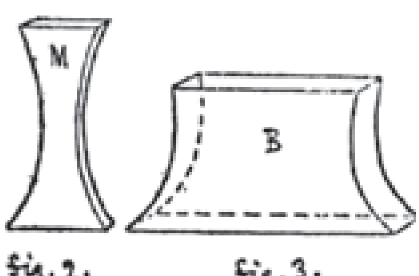


Figure 5. The realization of a submersible body and a tank with parallel sides according to Petrović's initial suggestion, figure taken from [Петровић 1898]

In case the input rotating cylinder D is grooved with the function $\eta=f(\zeta)$ according to which a body is immersed, then $x=\eta=f(\zeta)$ the level of the water y as a function of ζ brings the solution of the equation as follows

$$\alpha\phi(y)\frac{dy}{dx} = \beta\theta(f(\zeta)-y)f'(\zeta).$$

The solution is traced with a pencil as a graph on the output cylinder E.

Petrović asserts in (Петровић 1898) that the same device could integrate different equations by changing the shape of the tank and the immersed body and by adding several different tanks and integral cylinders, which resulted in the construction as the

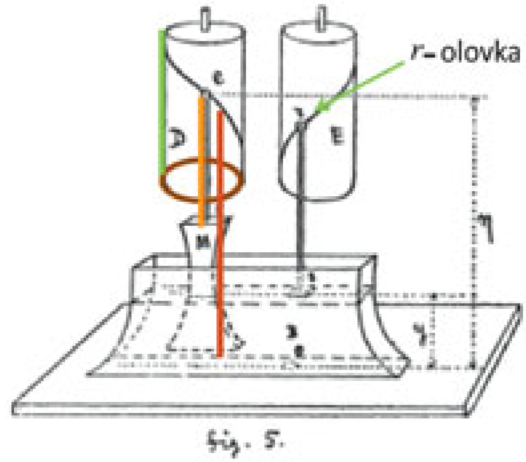


Figure 6. The principle of work of a perfected version of hydrointegrator, by adding the rotating input cylinder, figure taken from [Петровић 1898]

one shown in Figure 2, with the aim of enabling a simple replacement of a tank and body as the basic elements of the arithmetical unit of the device, placed on the outside in order to provide easy access.

As Petrović showed in (Петровић 1898), by selecting the body and tank of different shapes, as well as setting up other parameters, such as for example, the manner of immersing the body, equation (1) becomes a general model that incorporates five specific differential equations, including Riccati's equations

$$\frac{dy}{dx} = X(t) - \lambda^2 y^2$$

in whose methods of solving Petrović took a special interest, as well as the equation

$$\frac{dy}{dx} + F(y) = F(y)\psi(x),$$

$$\Phi(y)\frac{dy}{dx} + \lambda\sqrt{y} - af'(x) = 0,$$

$$\Phi(y)\frac{dy}{dx} = k(f(x) - y)f'(x),$$

$$\frac{dy}{dx} = f(y)\psi(ax + y).$$

Moreover, for another set of parameters and entry data, the hydrointegrator could also solve the equations such as the following

$$P(x,y)dx + Q(x,y)dy = 0. \tag{2}$$

Class (2) contains equations such as

$$\psi(x-y+\lambda)dx+(\varphi(y)-\psi(x-y+\lambda))dy=0,$$

$$f(y)dy+\psi(z)dz=0.$$

A detailed account of the aforementioned assertions related to the types of equations which could be solved by the hydrointegrator through selecting the manner of immersing, the shape of the body and tank and adding an orifice at the bottom of the tank, are presented by Petrović in (Петровић 1898; Petrović 1899a; 1899b).

At the time when Petrović was working on the hydrointegrator, it was the first machine for integrating differential equations that ran on the principle of hydraulics. A significant advantage of this compared to other devices of similar kind lied in the possibility of solving a greater number of equations by recombining its constructive elements. One of the existing integrators at the time, the Jacob's integrator, is based on some other principle, and could only solve Riccati's equation of the first order.

In (Петровић 1898) Petrović carried out a comparison of the proposed hydrointegrator with the corresponding devices with similar application, although based on different principles, presented in the catalogue of computing machines of 1892–1983 (von Dyck 1892). Figure 7 shows an extract from (Петровић 1898) in which Mihailo Petrović summarizes the advantages of his hydrointegrator.

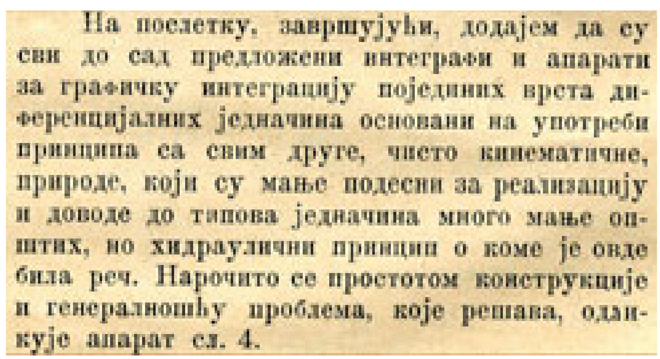

На послетку, завршујући, додајем да су сви до сад предложени интеграфи и апарати за графичку интеграцију појединих врста диференцијалних једначина основани на употреби принципа са свим друге, чисто кинематичне, природе, који су мање подесни за реализацију и доводе до типова једначина много мање општих, но хидраулични принцип о коме је овде била реч. Нарочито се простотом конструкције и генералношћу проблема, које решава, одликује апарат сл. 4.

Figure 7. A segment of text from (Петровић 1898) which refers to the comparison of the hydrointegrator with similar devices from that era

SCIENTIFIC PROMOTION OF THE HYDROINTEGRATOR

At the World Exposition of 1900 in Paris, France, at the Serbian Pavilion designed by the architect Milan Kapetanović, professor of geometry at the Technical Faculty of the Belgrade Grand School, Serbia presented the hydrointegrator of Mihailo Petrović among other representative exhibits, as an example of its scientific achievements. A prototype of the hydrointegrator was constructed for that occasion, for which a French constructor of precise mechanisms had been hired. There are no records of the name of that constructor, but he is mentioned in the letter of Mihailo Petrović addressed to the Minister of National Economy, in which he requests financial support for the realization of this device (Трифуновић 1968a). In that letter Petrović explicitly states that the working principle of the device was published in *Comptes rendus de l'Académie des Sciences de Paris*. At the said World Exposition, the hydrointegrator was awarded with a gold medal.

It is significant to note that at the time of the World Exposition, from 6 to 12 August, 1900, the International Congress of Mathematicians was also held in Paris, with Mihailo Petrović participating for the first time at a mathematical congress. As the prototype of the hydrointegrator had been made up to full functionality, it is reasonable to assume that the specialist public was familiar with this unique machine for integrating differential equations. As far as we know, it was the only model of the hydrointegrator that had been physically realized. The reconstruction of the hydrointegrator was carried out by Professor Dragan Trifunović in 1980, with the assistance of the architect Gradimir Bosnić. This reconstruction is kept at the Mathematics Department of the Faculty of Forestry in Belgrade.

The second international acknowledgement for the development of the hydrointegrator that Mihailo Petrović received was by the Mathematical Society of London, which conferred on him an honorary diploma in 1907 (Трифуновић 1968a).

CONTRIBUTIONS OF MIHAILO PETROVIĆ TO BUILDING ANALOGUE COMPUTING MACHINES

It has been widely accepted that the hydrointegrator of Mihailo Petrović is the first analogue computing machine based on the hydraulic principle, with particular emphasis on the fact that the same device had the capacity to integrate several differential equations (de Morin 1913; Митриновић 1955, 1958, 1960; Price 1900; Трифуновић 1968а; Willers 1949). Furthermore, the work on building the hydrointegrator is considered to be one of the most cited mathematical results of Mihailo Petrović.

The original contributions of Mihailo Petrović can be summarized as follows:

1. The application of hydraulic principle for solving differential equations;
2. The application of calculating elements in the form of immersed bodies;
3. The combination of the hydraulic principle and kinematic connection between the input and output rotating cylinders; and
4. Considerations related to the analogies between analytical facts for curved line integrals and geometrical facts from the theory of minimal surfaces with capillary phenomena (Петровић 1911). The analysis of these analogies would serve for building the hydrointegrator with several chambers in the arithmetical unit equipped with capillary tubes.

It should be noted that such a solution as the one with capillary tubes had been applied in the hydrointegrator of 1936 for solving Fourier's partial differential equation. This hydrointegrator designed by Lukyanov works on the principle of analogy between heat conduction and the model of fluid flow in capillary tubes (Lukyanov 1937, 1939).

More details about Petrović's work on the hydrointegrator can be found in (Stanković 2004; Трифуновић 1968а, 1968б, 1982).

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