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The Fractal Nature Approach in Ceramics Materials and Discrete Field Simulation

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Abstract:

In this paper, we present experimental data of static fractals and compare the results with theoretical analysis obtained using dynamic particle simulation. The fractal simulator presented in this manuscript is pioneer work and it is the base of the future concrete and industrial applications. We have examined sintered ceramic samples formed using several different additives, as fabricated using various parameters, sintering temperature and time. SEM analyses were performed on samples as a part of the experimental characterization procedure. Based on microstructures, sets of points have been selected as a primary database input for the theoretical-experimental simulation to model the processes that describe the experiment. For all grain and pore analyses, the fractal nature is recognized as a matter of substantial influence on material characteristics. All of our experimental and theoretical-experimental procedures are based on the construct that reconstruction of the grain and pore fractal nature is of enormous importance for microstructure property prognoses. The method presented here can be used to simplify and simulate, in real time, the interaction of a few thousand electrically charged particles possessing different masses through formulations based on Maxwell's electromagnetic equations. Particles in simulation interact with alternating (or static) electromagnetic fields and with static ceramics surface at the same time. All values are treated numerically. The fractal simulator consists of two components, a structure fractal generator, and field simulator. The functions for particle motion can be implemented and changed within the program in real time. The algorithm is written in the Delphi programming environment. The main result of this paper describes a quite new approach in the analysis of material microstructure properties towards programming-prognoses of the final properties of ceramic materials using the fractal nature within the fractal field simulator that generates structures, grains, and pores. The new simulator algorithm is developed as the important tool for the realization of the much ambitious project – simulation and realization of the Tesla's Fountain in ceramics. The concrete results will follow with this project realization in near future.

Keywords: *Ceramics; Sintering; Consolidation; Microstructure; Fractals; Computer simulation.*

1. Introduction

Presented in the paper are the basic experimental-theoretical settings and practical realization and usage of a simulator that simultaneously generates static structures through

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fractals and simulates dynamics of particles using the principles of quantum mechanics. As inspired by physical processes of nature, this dual approach provides an effort toward describing the simultaneous processes of synthesis and entropy that occur during processing of materials. Here, we experimental and theoretical analyze $BaTiO_3$ – ceramic samples possessing different additives and concentrations as processed using various conditions, pressing pressures (80 MPa-150 MPa) and sintering temperatures (1180-1380 ^oC) and as fabricated using the PTX–GASBARRE processing-technological line. The samples consolidation was performed by Murata.

Realization of generating static structures using fractals is based on theoretical studies of fractal geometry implemented in an efficient and optimal way in our simulator. The fractal structures generator can be directly changed and adjusted in the program, and thus may be adapted to address a wide range of physical processing that occurs during the fabrication and processing of materials. Simultaneously, the dynamic simulator algorithm as based on physics and field theory, using Maxwell's equations involving electrostatic and magnetic fields, provides an effective efficient tool to simulate particle dynamics. There are no apparent obstacles for implementing additional electromagnetic equations or other physical laws like gravity within the simulation formality.

Computer simulation is also quite an interesting branch of research. The basic idea behind our computer simulation is the development of efficient algorithms for numeric field calculations, using Maxwell's fundamental equations, for simulation of real processes occurring during interaction with electromagnetic fields and within the microstructure [1, 2]. One of the major obstacles for simulations is the number of the particles or quanta can significantly impact the complexity and time of simulating the interactions between electromagnetic fields and electrical charged particles or the interaction of other fields with quanta; thereby, resulting in the necessity to typically significantly limit the number of quanta used in simulations. While our research is based on such well-known theoretical and practical assumptions, in our approach we are able to go a step further using our simulation method which is able to handle up to 10000 particles or quanta in real time. Also, the system is able to provide visualization of motion and processes that particles undergo in real time; thereby, making the approach presented here useful for practical investigation of particle dynamics. The main advantage of the discussed numeric and computer approach is the fact that basic fields and other parameters can be altered efficiently within the algorithm which results in new input characteristics being calculated and presented in real time. To obtain similar results using classical methods [2], the method of managing similar empiric results becomes complicated and difficult if not impossible to accomplish in real time. In this paper, we present theoretical and practical aspects of this original computer quantum simulator methodology. The methodology, as applied through an algorithm using, is able to simulate the interaction of up to 10000 electrically charged particles in real time with interactions modeled based using Maxwell's electromagnetics equations. Calculations are performed by use of efficient algorithms. The engine is written in *Delphi 7* programming environment. The hardware used for simulations is Intel i7, 3.5 GHz, 16 Gb operating memory, with 500 Gb of SSD.

2. Analitical and Experimental methods

2.1. Euclidean vs. fractal models

The ceramic material investigations of V. V. Mitic and L. Kocic [6] with collaborators of ceramic materials, started roughly in 1996, focused on the relationships between Euclid and fractal aspect of ceramics using primarily barium-titanate microstructure. Some Euclidean models were investigated first. Such efforts have led us to extend Coble's model.

Sintered $BaTiO_3$-ceramics are characterized by extreme complexity due to the simultaneous and successive action of elemental mechanisms. Generally speaking, qualitative and quantitative descriptions are very difficult [1-3]. Ceramic grain contacts are essential for understanding complex electrodynamic properties of sintered materials [4-6]. Intergranular contacts are formed during the sintering process. When particles of barium-titanate powder form contacts, interatomic forces within the area of the contact interact to start forming a particle's neck and the aggregate may increase in density. Most of the sintering models have used Coble's two-sphere model [1] as the simplest model for studying elemental mechanisms responsible for the progress of the sintering process. Coble's model forms the basis for developing a multi-sphere model and the two-ellipsoid or multi-ellipsoid model which provides a better approximation of the sintering particles than the multi-sphere model [7,8]. For improved understanding of intergranular processes, Coble's model has been reconsidered using additional possible grains shapes (sphere-polyhedron, polyhedron-polyhedron). Special attention was paid to the neck growth modeling where it is supposed that the neck radius x can be written in the form $x = g(t;T,\mathbf{A})$, where T is sintering temperature, and $\mathbf{A}$ is system parameters vector. This vector depends on particle radius, diffusion coefficients, activation energies of transport mechanisms, boundary free energy, melting temperature, atom volume, and effective grain boundary thickness. All calculations are valid for the initial stage of the sintering process.

The sphere-polyhedron model was also considered. Among five regular polyhedra, an icosahedron is the best choice for two reasons: (1) it is the best approximation of the sphere; and (2) all its faces are triangles which simplifies the subdivision procedure. The next step was studying of polyhedron - polyhedron model. The importance of this model is in having a simple tool for manipulating, approximating and rapidly processing and evaluating the behavior of a large number of grains. Also, such a model can be used as a starting point for developing fractal geometry of intergrainular configuration.

The most complex is ellipsoid-ellipsoid model [12]. Grains of $BaTiO_3$-ceramic samples can be approximated using ellipsoids scattered throughout the material's volume. These ellipsoids can be used to model the behavior of grains in contact. Actually, due to sintering conditions of pressure, temperature, etc., grains partly penetrate into others, forming small contact regions that may be reasonably approximated as intersection of ellipsoids. Our aim is to determine the value of this contact region as a function of the grains' center distance.

Fractal models

The concept of "fractals" was developed by Mandelbrot, roughly 1965 to 1980, and "launched" in his seminal books [22, 23]. The new, *fractal geometry* left deep impression on the science community. According to Mandelbrot, a *fractal* is an object whose Hausdorff dimension, widely known as *fractal dimension*, strictly exceeds its geometric dimension. Fractal dimension is the main numeric characteristic of a fractal object [23]. Impressed by numerous demands of the growing group of specialists from different branches, Mandelbrot wrote [24]: *Fractals provide a workable new middle ground, between the excessive geometric order of Euclid and the geometric chaos of roughness and fragmentation.* Studying crystals, polycrystalline and amorphous materials using fractal analysis has revealed the dependence of the material's surface roughness over many orders of magnitude with its fractal dimension (Pfeifer [25] and Kaye [26]). The unit size used for measure is from the molecular level up. Many experiments have shown that a large scale of natural and artificial materials possess fractal structure. Some biological examples (broccoli, human brain, lungs tissue, neurons) stay side by side with "ideal" or "mathematical" fractals like the Cantor set, Gosper contour, Koch curve, Sierpinski triangle/square etc.

Becoming aware of the crucial importance of fractal properties of ceramics materials, Prof. Mitic's group performed their first measurements of fractal dimension of the contour of

$BaTiO_3$-ceramic- grain- profiles and obtained the interval 1.06 <$DH_{contour}$< 1.08 [30]. Next, they determined the fractal dimension of the surface of $BaTiO_3$-ceramic- grains [31] and obtained the result 2.010 <$DH_{grain\ surface}$< 2.085. Finally, the result 2.7529 <DH_{BaTiO3}<2.8025 have been obtained for the surface of ceramic bulk [32]. These results were obtained for barium-titanate ceramics without additives. Of course, using different dopants in the processing of ceramics provide varying results depending on the dopant and dopant concentration.

2.2. Experimental procedure

Samples were prepared using commercially available $BaTiO_3$ powder (MURATA) and reagent grade powders (Fluka chemika). For the research, pure $BaTiO_3$, as well as a powder based on $BaTiO_3$, with various additives (CeO_2, $MnCO_3$, Bi_2O_3, Fe_2O_3, $CaZr_2O_3$, Nb_2O_5, Er_2O_3, Yb_2O_3, Ho_2O_3, La_2O_3, Sm_2O_3, Dy_2O_3) with concentrations from 0.01 up to 1 wt%. The complete experimental procedure is shown in Fig. 1.

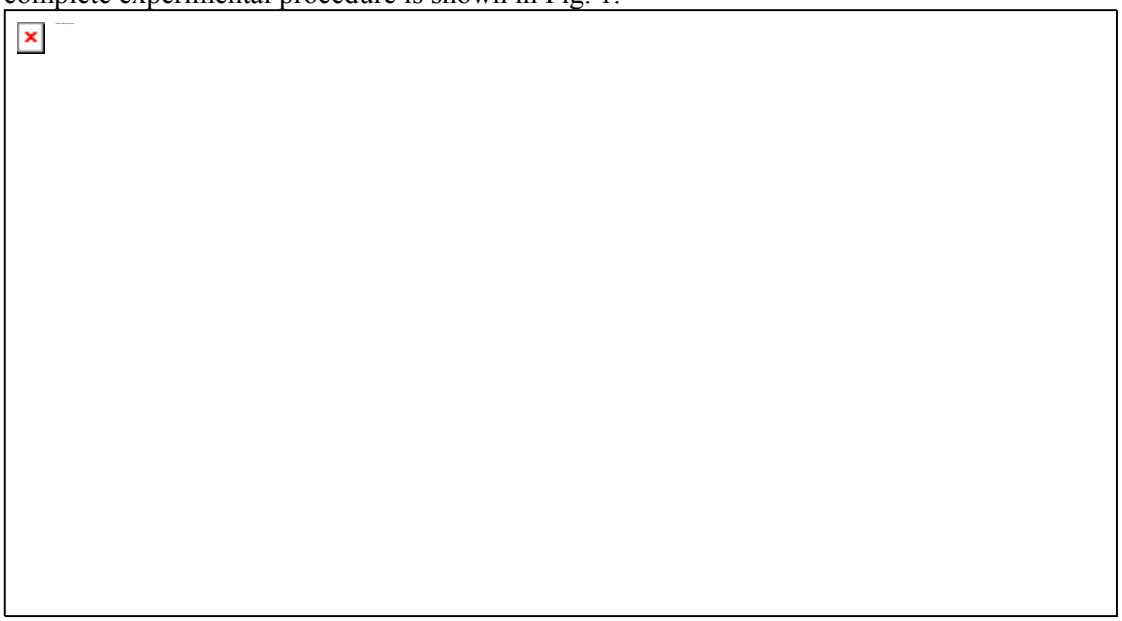


Fig. 1. Ceramic body preparation manufacturing process.

The pressing pressure, sintering conditions including temperature and time, and the additive concentration content with respect to the fractal nature of the microstructure $BaTiO_3$-ceramics have been investigated and characteristics.

After measuring pure $BaTiO_3$ and additives powders, the mixture was processed in a ball mill, in which a certain amount of water and an organic binder were added. Homogenization of the mixture was carried out for about 48 h. Mass was transported to the sprayer by membrane pump, where it was dried and determined powder granulation was obtained. Then, the material was collected in a special vessel, and its bulk density was investigated every hour. Occasionally, a granulation analysis was performed using a vibrating sieve, which was essential for pressing. The powder particles were spherical shape, size 10-130 μm, with the appearance of particle agglomerates. The powder preparation and processing is schematically illustrated in Fig. 2.

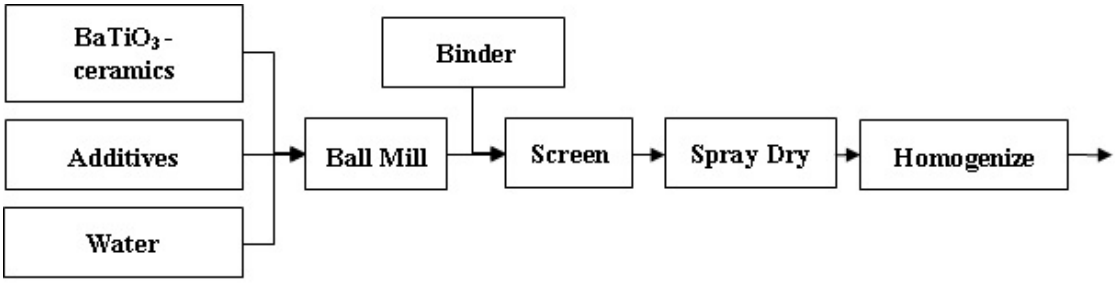


Fig. 2. $BaTiO_3$ powder consolidation.

In order to investigate the pressing pressure influence, i.e. the green density performs influence on the obtained sample characteristics, a pure $BaTiO_3$ sample of green density using double-sided pressing with pressures from 80 to 150 MPa were formed from $BaTiO_3$-ceramics based mixture with specific additives. The samples were formed using a hydraulic press (JAPAN KYOTO Murata B1) which is also applied for multi-layer capacitor technology. The sample square shape formed using a PENTRONIX – PTX – GASBARRE press (Fig. 3).

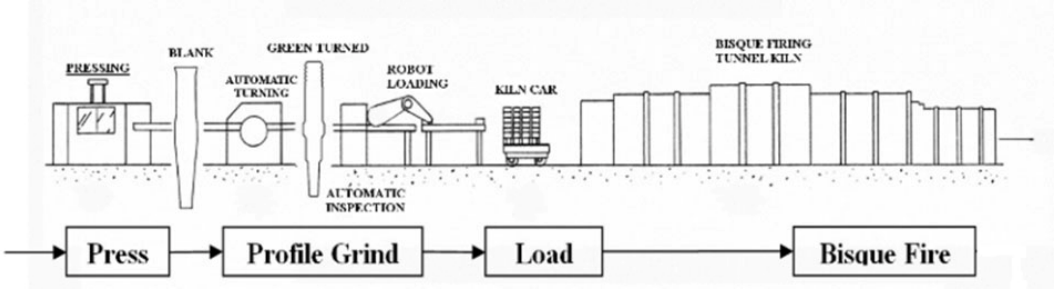


Fig. 3. $BaTiO_3$ sample processing pressure consolidation.

Sintering was performed in an electric tunnel furnace (CT-10 Murata) at temperatures from 1180 °C to 1380 °C and were sintered for two and four hours during the 38 h to complete the process (Fig. 4).

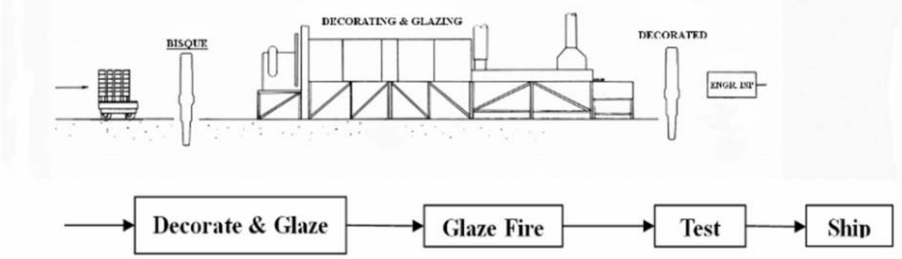
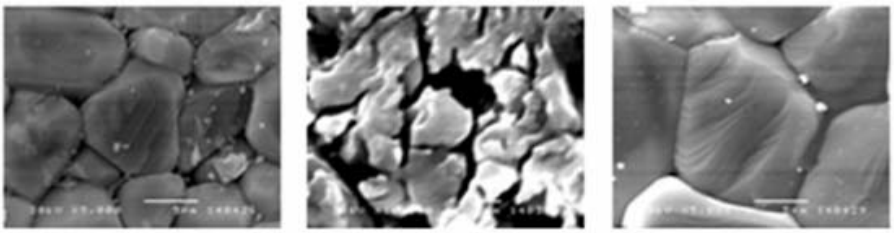


Fig. 4. CT-10 Murata.

The microstructures and compositional studies of sintered or chemically etched samples were observed using scanning electron microscope (JEOL-JSM 5300) equipped with energy dispersive x-ray analysis spectrometer (EDS-QX 2000S system). The grain size and porosity distribution of samples were obtained using LEICA Q500MC Image Processing and Analysis System. The linear intercept measurement method has been used to estimate the grain size values, as well as the pore volume ratio. It was necessary to prepare a series of samples based on $BaTiO_3$-ceramics to further elucidate the relations among fractals, microstructure and electrical properties. The pressing pressure, sintering conditions including temperature and time), and the additive concentration content with respect to the fractal nature of the microstructure $BaTiO_3$-ceramics have been investigated and characteristics.

In the early research and development stages, a small measuring device Run-meter, which measures the contour perimeter in a given image plane, is utilized for the box-counting method to retrieve the fractal dimension, DHf, which is the most important quantity involved with the notion of fractals (by Mandelbrot). The fractal dimension, which is a natural extension of the usual notion of dimension, - is usually called the topological or geometric dimension. The obtained perimeter data, for at least five different magnifications of the same observed grain, can be applied to the Richardson' law and to further grain shape fractal reconstruction - designing procedure.

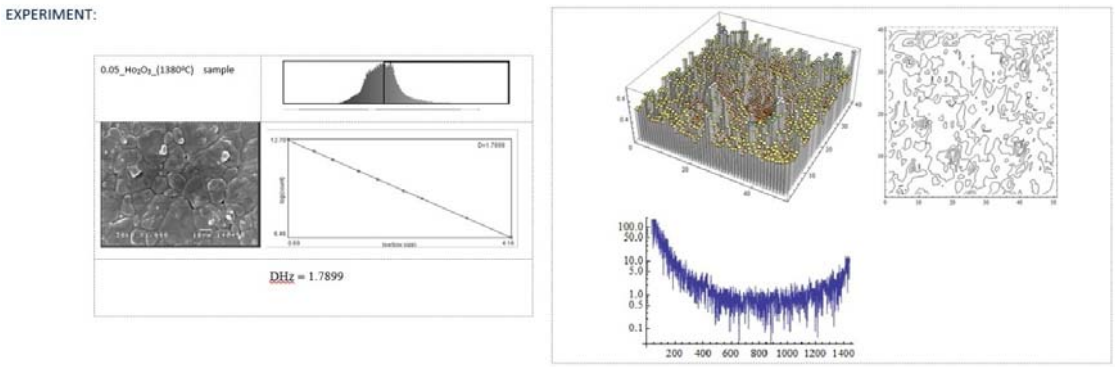


a) b) c)

Fig. 5. SEM images of Ho_2O_3 doped $BaTiO_3$ a) 0.01 wt%, 1320 ºC, b) 0.01 wt%, 1350 ºC, c) 0.01 wt%, 1380 ºC.

Each of these micrographs, which is obtained with the SEM microscope in digitized form, typically in a 1808x1440 resolution in an 8-bit gray scale, could be further processed in some graphical editor (Fig. 5). The selected ceramic grain's contour transfers into analytic form applying fractal interpolation procedure introduced by Barnsley [33, 34]. After perimeter point sample acquisition, the fractal interpolation is applied. The result is a set of piecewise linear parametric functions defined by an iterative procedure.

It should be mentioned that all these steps, which have the microstructure analysis experimental character obtained by microscope (Fig. 6), as well as grains and pores perimeters data collection, are, in fact, part of synthesis of a completely new ceramic materials, and materials in general. This is a characterization method innovation that enables connection between ceramic microstructures and fractal analyses methods towards the fractal simulator development. The fractal nature gives much more natural approximation to real physical structures then classic Euclidian geometry [34]. The fractals are not just geometric elements. This means that processing parameters may have fractal behavior in time, for ex. change of energy, motion of particles (Brownian walk), variation of electric or mechanic parameters etc. Such kinds of fractal behavior are also supported in our particle simulator. The fractal dimension offers a mighty instrument for fine distinction and precise identification of processes that may look identical at the first sight but may not be identical.



a) b)

Fig. 6. Surface fragment (experiment) a) Fractal dimension estimation, b) Surface elements of the previous sample and its Discrete Fourier Transform.

3. Static fractal structure generation

In this Section we will discuss the static portion of the simulator based on fractals. Fractal approach in static representation of material structure as well as fractal correction is a new approach. This part of simulator is based on efficient algorithm for fractal design.

Julia sets

Let $f(z) = P(z)/Q(z)$ be a rational function, where $z \in C^*$ ($C^* \cup \{\infty\}$ is the Riemann sphere), and P and Q are polynomials without common divisors. The "filled-in" Julia set F_f is the set of points z which do not approach infinity after $f(z)$ is repeatedly applied. The true Julia set J_f is the boundary of the filled-in set (Fig. 7).

The key importance for our work is the fact (proven in Complex analysis) that any closed planar subset can be approximated in Hausdorff metrics by the Julia set of a rational map. This means that any 2D projection of a ceramic grain can be represented by a Julia set. Similarly the construct may be extended to 3D space, with the only difference being that instead of complex numbers, hypercomplex numbers (or quaternions) are used.

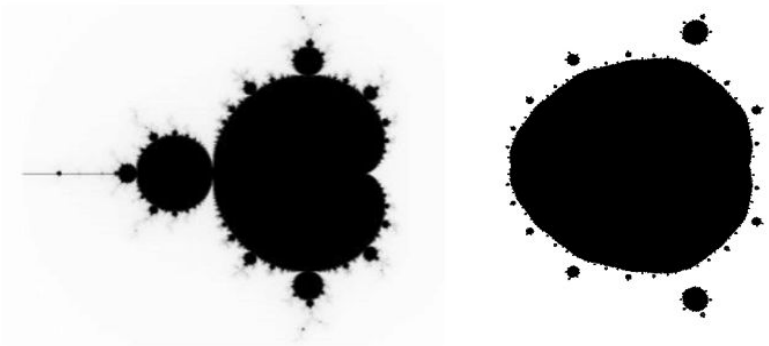


Fig. 7. Mandelbrot fractal set (left) and Julia set (right).

Mandelbrot sets

The Mandelbrot set is obtained from the quadratic recurrence equation:

$$Z_{n+1} = Z_n^2 + C \quad (1)$$

with $Z_0 = C$, where points C in the complex plane, for which the orbit of Z_n does not tend to infinity, are in the set. Setting Z_0 equal to any point in the set that is not a periodic point gives the same result. The Mandelbrot set was originally called a *micro* molecule by Mandelbrot. J. Hubbard and A. Douady proved that the Mandelbrot set is connected. What is the role of Mandelbrot set if it is quoted a few lines above that Julia sets are used for approximation of real grains? The answer is simple. The Mandelbrot set represents the "catalogue" of endless quantity of Julia sets. So, it contains "addresses" of all Julia sets of a kind. More specific detail about Julia and Mandelbrot can be found in several articles [22, 23, 33].

Real-time fractal simulator for dynamic processes and static material structures

During the production processes, which are presented in this paper, structures and materials are formed by the method of sintering. A number of parameters such as the density of the consolidated material and spatial layout of the components are very complex fractal functions, which depend equally on the process itself, as well as on the structure and the

composition of the material. Physical interdependences are also very complex and partially unknown. Because of these reasons, model assembling, which will have the ability of the process prediction, is also extremely complex.

Having the goal of simplifying the model in mind to a tractable level, the approach of the model reduction to 2D grid is used, and elementary particles themselves (that can also be dust particles) are modeled in a way so that they can initially be described using any of a variety of functions, even *Boltzmann's* model of distribution of the energy at certain points, too. These elements are defined by the program, as well as the unit memory slot of the RECORD type, which contains all of the physical characteristics necessary for the simulation. Thus, the simulation is reduced to a numerical approach and it can now enable matching and appropriate modeling of the local stochastic effects and processes.

Naturally, the accuracy of the simulation is defined by the initial resolution and above all depends on the capacity and ability of the computer, which will be used to launch the simulation. As it was already mentioned in this paper, the simulator is oriented to operate in real time and can be divided into two large subsystems – static (simulator of the material) and dynamic (simulator of motion of charged particles of different masses and charges) in stationary alternating variable electric and magnetic fields.

The static part of the simulator, which builds the structure in 2D simulation space, is based on data which can be predefined or dynamically generated. In case there are results and precise mapping of the surface from for instance electron microscopy, such data can be directly downloaded as default values into memory matrix of the simulator. Simulator operating with the structure, which is analogous to that in the nature, is achieved through such a positive feedback loop. In case the dynamic generator is used, the simulator uses fractals in order to describe the final structures gained by material cooling in accordance with the results showing that the fractal structure can be applied in order to more precisely define those processes which are responsible for forming the final material structure. During the operating process, two basic types of the fractals were used (Mendelbrot and Julia) and their mathematical definition is given in the introductory sections.

The dynamic part of the simulator is designed for the initiation of a large number of charged particles in real time in 2D space of the material, in order to simulate the movement under the influence of electric and magnetic fields in already consolidated material. In basic operating mode of the simulator, when the fields are turned off, the particles are moving under the rules of the *Brownian motion* following *Gauss'* energy distribution. Note that the fractal dimension of Brownian motion in plane is 2, while in 3D space is 3. In other words, the path of 2D/3D Brownian motion fills the plane/the space.

After turning on magnetic and electric fields, which utilize the superposition principle, the particles begin to move under the rules defined by Maxwell's equations and to interact with already formed material. Having in mind the basic resolution of the particles of the same order of magnitude, as well as the elementary particles, which form a static grid, the simulator can operate on any predefined resolution level – subatomic, ion-atomic, or on the level of the elementary micro-dust.

Therefore, we have not only consolidated modeling approach through the fractals, we also have the possibility to observe the complex processes which appear in consolidated materials during the particle movement, with or without electromagnetic fields, by using the simulator, too. We consider such ability as one of the basic novelties presented in this paper.

Our approach of fractal generation, the core iterative processes, are presented in the lines of code, Appendix I. Fractals, which arise due to stochastic processes, can be generated using standard randomize functions. After full number of points explored in space, fractal is completed (Fig. 8, Fig. 9).

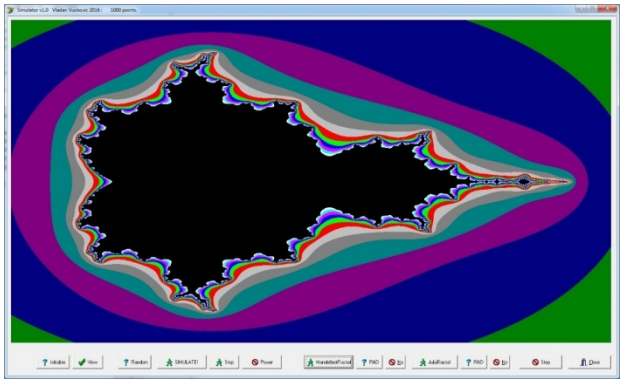


Fig. 8. Mendelbrot fractal solution.

In a similar manner, Julia fractals can be generated.

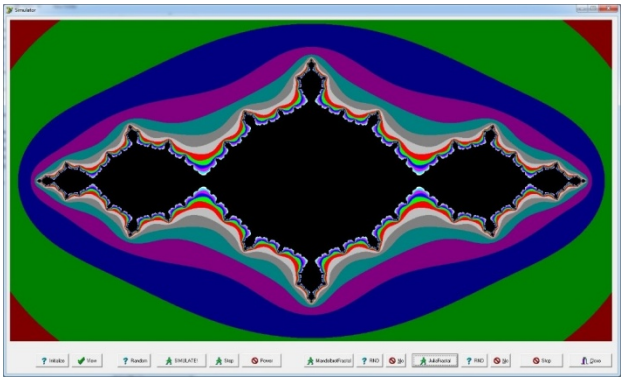


Fig. 9. Julia fractal solution.

3.1. Dynamic Particle Simulation

Our algorithm has been developed in *Delphi* (*Pascal for Windows*). Our choice of programming language is influenced by the clarity and strong mathematical background with *Pascal*, as well with solid performances within *Windows* environment. Nevertheless, it is not complicated to move the whole application to C++. We will present the basic data definition for the application here. The main idea is that the complete status for all particles can be held in one record array. The record type can be defined in the following way (Appendix II). Using that basic data structure, all calculations are performed on every quantum separately, using simple numeric forms. For instance, the electrostatic calculations are performed in one pass (Appendix III). Also, calculation for magnetic field is performed in one pass (Appendix IV). This procedure uses data stored in basic strings (*type_elec = record).* The procedure controls display bounds, animates particle movement, etc.

3.2. The Problem of Particle Rotation in 3D Virtual Space

One very important problem that needs solution is how to simulate three-dimensional particle moving in space. The solution must be efficient enough to support real time operations with thousands of particles. This request implies minimal use of trigonometrical (sinusoidal) functions. One of the original solutions used in our algorithm is based on different mathematical trigonometrical transformations. The next procedure is very efficient and can be used to generate tree dimensional movements for a large amount of particles in real time. The procedure is written in *Pascal* programming language (Appendix V). This

procedure re-calculates all three coordinates from the data arrays in one pass. The procedure is extremely efficient by using temporal variables for storage of the trigonometrical function results.

3.3. The Algorithm

The algorithm is completely written in *Delphi 7* for *Windows*. The main idea behind this algorithm is that user has complete control of all aspects of the program, choosing options in real time. The user can control different aspects of the simulator:

- Number of particles (up to 10000),
- Electric charge and mass of each particle,
- Electrostatic and magnetic field strength,
- Alternating and direct components of electric charge on the borders of the simulator space,
- Step by step simulation,
- Continual simulation *(infinity loop*).

This kind of algorithm is very useful for simulating the dynamic movements of charged particles in electromagnetic fields in vacuum. Also, another interesting opportunity for use of the algorithm is investigation of the charged particle movements inside a condenser. We have optimized the algorithm for particle simulation for our particular investigations (Fig. 10).

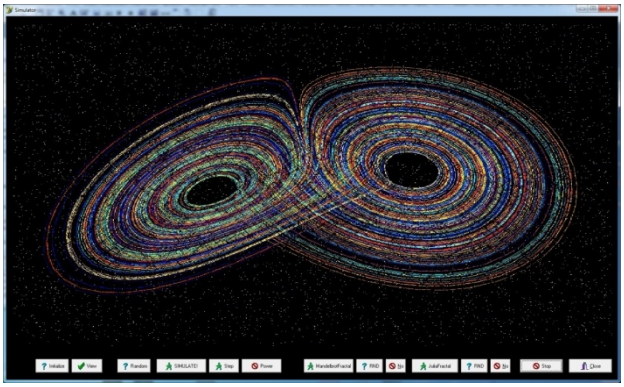


Fig. 10. Particle simulator engine with Julia sets orbital.

In this test, the algorithm is operating on 10000 objects being sintered together using two types of particles. If no static fractal is presented, the algorithm acts like *Brown* motion generator.

3.4. The Merging of Static and Dynamic Simulation

By integrating the static fractal approach within the dynamic particle algorithm, we can simulate processes similar to natural ones. The outcome is very similar to erosion processes. If we use different setup for magnetic and electric field, we can generate particle erosion of fractals. The process is similar to the nature processes, where fluid flow generates particle erosion.

4. Conclusion

The fractal and quantum approach in computer modelling of the complex structures is not well known nor well explored. Some important novelties in this paper are development of

the new combined static (fractal) and dynamic (particle) algorithm by the author that is able to handle a few thousands of particles in real time.

Ceramics materials are outcomes of technological processes of consolidation of powder-based input. During this process, material changes its inner structure that can be described, from the fractal analysis point of view, as fractal dimension increasing of the ceramic bodies. The final fractal dimension for $BaTiO_3$ ceramics reaches the value 2.75 to 2.9, compared with the theoretic limit, value of 3. So, the aim of this study is to apply fractal analysis on transformation of ceramic materials during sintering processes that have both static and dynamic components. The idea is to obtain the static model using Julia fractal sets to approximate ceramic grains, using the complex functions over the complex plane (2D images) or over the quaternion field (for 3D bodies).

Next, the dynamic process is simulated by the Brownian motion, ruled by Lorentz force law and Gauss energy distribution, and taking care of electromagnetic field through Maxwell partial equations. This approach reproduces particle dynamics during liquid sintering stage and provides control over its dynamics.

Combination of presented static and dynamic simulations may be useful in further study of processes emerging during ceramics consolidation that may lead to new frontiers for deeper and higher level electronics circuit microelectronic integration, and the new frame of fractal electronics.

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APPENDIX I

```
...
begin
   Xcord := -0.8 + 3 * Li
   Ycord := -1.4 + 2.8 * Ji
   color := 0;    W := 0;    K := 0;
   repeat       { The core iterative process. Numeric fractal generator}
     Z := SQR(W)-SQR(K)-Xcord;
     V := 2*W*K-Ycord;
     U := Z;
     Color:=Color+1;
   until (SQR(W)+SQR(K)>9) or (Color=14);  {Color depth}
end;
end;
```

APPENDIX II

```
...
type type_e = record
          x,y:real;              {x and y coordinates}
               vx,vy:real;           {Real velocities}
          newx,newy:integer;  {New virtual coordinates}
          oldx,oldy:integer;  {Old virtual coordinates}
          q,m:real;              {Mass and electric charge}
          mass:integer;
end;
```

APPENDIX III

```
...
procedure electro_field_numeric;
var Dvx,Dvy:real;
begin
   for i:=1 to num do
   begin
      Dvy:=t[i].q*(EY+ESTAT)*Dt/t[i].m;   {Direct implementation of Maxwell law}
      t[i].vy:=t[i].vy+Dvy;
   end; end;
```

APPENDIX IV

```
...
```

```
procedure MOVEt;
var rx,ry,D:real;
begin
   for i:=1 to num do
   begin
      rx:=coord[i].vx*deltat;
      ry:=coord[i].vy*deltat;
      coord[i].x:=coord[i].x + rx;      {Move to x axis}
      coord[i].y:=coord[i].y + ry;      {Move to y axis}
      if (coord[i].x<1) then begin coord[i].x:=MX-1; end; {x left limit}
      if (coord[i].x>MX) then begin coord[i].x:=1; end; {x right limit}
         if (coord[i].y<=1) then begin coord[i].vy:=-(coord[i].vy/3); coord[i].y:=coord[i].y -
ry; end;  {y control}
         if (coord[i].y>=MY-1) then
               begin
                 coord[i].vy:=-(coord[i].vy/3);
                 coord[i].y:=coord[i].y - ry;
                 D:=random(speed_reduction);
                 D:=D+1;
                 coord[i].vx:=coord[i].vx/D;
               end;
   end;
end;
```

APPENDIX V

```
...
procedure ROTOR (xN,yN,zN,zooming:real);
var x,y,z,xt,yt,zt:real;     {Temporal X,Y,Y coordinates }
sin_xN,cos_xN,sin_yN,cos_yN,sin_zN,cos_zN:real; {Trigonometrical}
    f,sx,sy,zz: integer;
    ot:word;
begin
max:=-24000;

{Trigonometrically transformations of particle angles in all three axes}

sin_xN:=sin(xN);cos_xN:=cos(xN);
sin_yN:=sin(yN);cos_yN:=cos(yN);
sin_zN:=sin(zN);cos_zN:=cos(zN);
for f:=1 to points_num do
begin
x:=px^[f];
y:=py^[f];
z:=pz^[f];
Yt:= Y*COS_XN - Z*SIN_XN;    {Calculation of the position after rotation}
Zt:= Y*SIN_XN + Z*COS_XN;   {Move around x axis}
Y:= Yt;
Z:= Zt;
Xt:= X*COS_YN - Z*SIN_YN;  {Move around y axis}
Zt:= X*SIN_YN + Z*COS_YN;
X:= Xt;
Xt:= X*COS_ZN - Y*SIN_ZN;   {Move around z axis}
```

```
Yt:= X*SIN_ZN + Y*COS_ZN;
px^[f]:=xt;
py^[f]:=yt;
pz^[f]:=zt;
pxx^[f]:=round(zooming*x+300);    {Zoom-ing factor}
pyy^[f]:=round(zooming*y+200);
zz:=round(zoom*z);
if zz>max then max:=zz;
pdz^[f]:=zz;
end
end;
```

Садржај: *У овом раду ћемо представити експерименталне податке статичких фрактала и упоредити их са теоријским анализама добијеним коришћењем динамичких честичних симулатора. Фрактални симулатор који је приказан у раду је пионирски рад и основа будућих конкретних индустријских апликација. Користили смо синтероване керамичке узорке који су консолидовани са различитим додацима и параметрима, као што су температура и време синтеровања. Такође, изведене су СЕМ анализе као део експерименталне процедуре за анализу и карактеризацију. Узимајући микроструре за основу, одабран је сет полазних тачака као примарна база података за теоријско-експерименталну симулацију процеса моделовања. У свим овим анализама зрна и пора, природа фрактала је препозната као елемент од суштинског значаја. Све наше експерименталне и теоријско-експерименталне процедуре прожете су идејом о примени фрактала у реконструкцији зрна и пора, а то је и питање од огромног значаја за претпоставке микроструктуралних својстава. Метод који је приказан може се искористити да се симулира интеракција неколико хиљада наелектрисаних честица са различитим масама преко формула базираних на Максвеловим једначунама. Честице у симулатору интерагују са наизменичним (или статичким) електромагнетним пољем као и са статичком керамичком подлогом у исто време. Све вредности у симулацији третирају се нумерички. Симулатор фрактала се састоји од две компоненте, структуралног фракталног генератора и симулатора поља. Функције кретања честица могу бити промењене у реалном времену. Апликација је написана у DELPHI програмском окружењу. Најважнији резултат овог рада је потпуно нов програмски приступ својствима микроструктуре материјала и анализа према претпоставкама финалних својстава керамичких материјала у оквиру фракталне природе, уз помоћ симулатора генерисања структуре зрна и пора. Нови алгоритам за симулацију је развијен као значајни програмски алат за реализацију много амбициознијег пројекта – симулације и реализације Теслине Фонтане у керамици. Конкретни резултати ће уследити у блиској будућности.*

Кључне речи: *Керамика, синтеровање, консолидација, микроструктуре, фрактали, компјутерска симулација.*