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FLUCTUATIONS OF THE NUMBER OF ADSORBED MICRO/NANOPARTICLES IN SENSORS FOR MEASUREMENT OF PARTICLE CONCENTRATION IN AIR AND LIQUID ENVIRONMENTS*

Article Highlights

- Expression for the signal-to-noise ratio is derived considering adsorption-desorption fluctuations
- Mass transfer of target particles decreases signal-to-noise ratio (SNR) in environmental sensors
- Two orders of magnitude increase in SNR by elimination of the influence of mass transfer

Abstract

A theoretical model of fluctuations of the number of adsorbed micro/nanoparticles in environmental sensors operating in air and liquids is presented, taking into account the effects of the mass transfer processes of the target particles in a sensor reaction chamber. The expressions for the total power of the corresponding adsorption-desorption noise, and for the corresponding signal-to-noise ratio are also derived. The presented analysis shows that the transfer processes can have a significant influence on the sensors limiting performance. The influence on both the fluctuations spectrum and the signal-to-noise ratio is estimated at different values of target particles concentration, functionalization sites surface density, and adsorption and desorption rate constants (the values are chosen from the ranges corresponding to real conditions). The analysis provides the guidelines for optimization of sensor design and operating conditions for the given target substance and sensor functionalization, in order to decrease the influence of the mass transfer, thus improving the ultimate performance (e.g., minimal detectable signal, signal-to-noise ratio) of sensors for particle detection. The calculations we performed show that it is possible to increase the signal-to-noise ratio for as much as two orders of magnitude by using optimization that eliminates the mass transfer influence.

Keywords: environmental sensor, micro/nanoparticle, adsorption-based sensor, signal to noise ratio, mass transfer.

Monitoring of concentration levels of micro/nanoparticles in the air and liquid environments is very significant for environmental protection, public healthcare, agriculture and other fields. Different plat-

forms exist for detection of particles and measurement of their concentration in a sample [1]. The principle of operation of a large group of sensors is based on selective reversible adsorption of target particles on a functionalized sensor surface, occurring in the sensor's reaction chamber and causing a change of some of the measured parameters (mechanical, optical, electrical) [2,3]. In this paper we examine a class of adsorption-based sensors with a flow-through reaction chamber (e.g., plasmonic, FBAR - thin Film Bulk Acoustic wave Resonators, QCM - Quartz Crystal Microbalance, microcantilever sensors, etc. [4-6]),

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fabricated using micro- and/or nanotechnologies. They are proven to be highly sensitive to small analyte concentrations, have small dimensions, low-power consumption, low-cost and fast response, which enables their application as portable devices for *in situ* real-time measurements, and possible wireless connection for real-time remote access to monitoring data. They are considered not only as an alternative to expensive and bulky laboratory equipment, but also they offer a spectrum of new applications, e.g. measurement of personal exposure levels, sensor networks for *in situ* monitoring of the temporal and spatial variability of pollutant concentration etc. The value of the sensor's response is determined by the total number and mass of adsorbed particles, and its time evolution depends on the rates of adsorption and desorption, and also on the rates of transport processes (convection and diffusion) of target particles, by which they are moved through a microfluidic sensor system, to (or from) the immediate vicinity of the adsorption sites at the sensing surface [7]. Unavoidable fluctuations of the number of adsorbed particles cause fluctuations of the sensor's output signal, known as adsorption-desorption (AD) induced fluctuations, which affect the sensor's ultimate performance (noise, signal-to-noise ratio, minimal detectable concentration). As the dimensions of the sensing element decrease, which is the general trend in the field of micro/nanosensors, the contribution of AD fluctuations to the total sensor response fluctuations increases. The detailed derivation of the theoretic model of fluctuations of the number of adsorbed micro/nanoparticles in environmental sensors, which takes into account the mass transfer processes, is given in [8]. The approximations used in the derivation are applicable for transport-reaction regimes typical for the mentioned types of affinity-based sensors with a flow-through reaction chamber. A characteristic of such regimes is the formation of a thin layer depleted of target particles, adjacent to the functionalized sensor surface. Using the derived theory, a quantitative analysis of the mass transfer influence on the fluctuation process time constant was performed in [8].

In this paper we first shortly present the theoretic model of the fluctuations of the number of adsorbed particles, which are caused by the stochastic nature of AD processes coupled with transport processes of particles. The model is derived assuming the Langmuir adsorption of target particles, which is usually used for description of adsorption kinetics in surface-based sensors, since it matches experimental data in a majority of cases [6,7,9]. We then use the obtained expression for the spectral density of fluctuations

for determination of the corresponding noise (called the adsorption-desorption noise), and also of the sensor signal-to-(AD noise) ratio. We investigate the influence of the transport processes on the parameter of the spectral density of the fluctuations, known as "the plateau" value of the Lorentzian spectrum. Based on numerical calculations, the influence of the transport processes on the signal-to-(AD noise) ratio will be considered, as well as the dependences of that influence on the concentration of target particles, on the surface density of the functionalizing sites and on the affinity of target particles for the adsorption sites on the sensing surface. The presented theory and analysis enables estimation of the sensor's ultimate performance and provide the guidelines for their improvement (better signal-to-noise ratio and lower detection limits) through the optimization of experimental design parameters for the given type of particles whose concentration is within the range of interest.

THEORETICAL DERIVATIONS

In a many micro/nanosensors, the principle of operation is based on the change of a measured parameter caused by the adsorption of the target particles from the air or liquid environment. Therefore, the response of such sensors is determined by the number of adsorbed particles, $N(t)$. In the sensor flow-through reaction chamber the following processes occur: adsorption, desorption, convection and diffusion of target particles. Adsorption and desorption processes take place on the functionalized sensor surface. Adsorption can occur when a target particle is transported to the immediate vicinity of a surface adsorption site. A desorbed particle can be adsorbed again or moved away from the adsorption site by diffusion or convection.

The following assumptions are introduced: 1) the adsorption sites surface density is approximately uniform [7]; 2) only one target particle can be bound to any adsorption site; 3) all the adsorption sites are equivalent; 4) there is no interaction between target particles. They are in accordance with the Langmuir adsorption.

The typical transport-reaction regime in FBAR, QCM and plasmonic sensors with a flow-through reaction chamber [4,6,7,9] is such that a thin layer depleted of target particles forms adjacent to the functionalized surface, when the kinetics of the binding process is limited by diffusion (in the case of reaction-limited kinetics the concentration of target particles in a reaction chamber is uniform, so the problem is much simpler for mathematical analysis). In a dif-

fusion-limited case, the distribution of the particle concentration in a reaction chamber can be approximated by a model that assumes the existence of two regions (*i.e.*, compartments). In the first, narrow region adjacent to the sensing surface (the inner compartment), the concentration gradient exists, while in the second, encompassing the rest of the reaction chamber (the outer compartment), the concentration is uniform and equals the particle concentration in the analyzed sample, C . According to the two-compartment model it can be assumed that: 1) the imaginary boundary between the two compartments is parallel to the sensor surface; 2) the concentration of the target analyte in the chamber in the immediate vicinity of the sensor surface, C_s , is uniform; 3) for all the parameters of the adsorption-desorption and transport processes the values can be used that are obtained by averaging across the surface on which adsorption occurs. The suitability of the model for the analysis of the kinetics of the process of binding of particles to a functionalized surface was experimentally confirmed [7] and also by numerical simulations [10]. According to the model, the equations describing both the transport between the compartments and the AD process of target particles to the sensing surface are [7,8]:

$$k_m A(C - C_s) = dN / dt \quad (1)$$

$$dN / dt = k_f C_s (N_m - N) - k_r N = a_{eq}(N) - d_{eq}(N) \quad (2)$$

where N_m is the total number of adsorption sites on the functionalized surface of the area A , k_f and k_r are the adsorption and desorption rate constants, respectively, between the target particles and adsorption sites, k_m is the mass transfer coefficient of target particles, while $a_{eq}(N)$ and $d_{eq}(N)$ denote the equivalent rate of the processes that contribute to the increase of the number of adsorbed particles and the equivalent rate of the processes that decrease the number of bound particles, respectively. Eq. (1) is obtained from the condition of mass balance at the surface (diffusive flux equals the rate of change of the adsorbed amount) in a diffusion-limited case [7,9], while Eq. (2) describes the Langmuir adsorption kinetics.

The number of adsorbed particles on the sensor's functionalized surface, $N(t)$, randomly fluctuates because it is a result of stochastic AD processes coupled with transport processes. Let us observe the fluctuations ΔN around the expected equilibrium value, N_e , assuming $\Delta N \ll N_e$. Based on Eqs. (1) and (2), a nonlinear differential equation is obtained for N . Its linearization around N_e yields the fluctuation equation:

$$d(\Delta N) / dt = \left(da_{eq} / dN - dd_{eq} / dN \right) \Big|_{N=N_e} \Delta N = -\Delta N / \tau \quad (3)$$

$$\tau = \frac{1}{k_r + k_f C} \left(1 + \frac{k_f k_r N_m}{k_r + k_f C} \frac{1}{k_m A} \right) \quad (4)$$

$$N_e = N_m k_f C / (k_r + k_f C). \quad (5)$$

Equation (3) assumes the Langevin form after addition of the stochastic term η on the right side. According to [8], based on the Langevin equation the expression is derived for the single-sided power spectral density of the fluctuation process:

$$\begin{aligned} S_{\Delta N}(\omega) &= S_\eta(\omega) \tau^2 / (1 + \omega^2 \tau^2) = \\ &= 4a_{eq}(N_e) \tau^2 / (1 + \omega^2 \tau^2) = \\ &= 4k_f N_e \tau^2 / (1 + \omega^2 \tau^2) \end{aligned} \quad (6)$$

which is of the Lorentzian type ($\omega = 2\pi f$). The spectral density of the number of adsorbed particles equals $(S_{\Delta N}(f))^{1/2}$ ($1/\text{Hz}^{1/2}$), and the parameters by which it is fully determined are the spectral density for $f \rightarrow 0$, the so-called plateau, P , and the frequency at which the spectral density decreases by the factor $2^{1/2}$ (the cut-off frequency). This frequency equals $1/(2\pi\tau)$, where τ is given by Eq. (4). Based on Eq. (6), the plateau is determined by:

$$P = \sqrt{4a_{eq}(N_e) \tau^2} = 2\tau \sqrt{k_f N_e}. \quad (7)$$

By using Eqs. (4) and (7) the spectral density of fluctuations of the number of adsorbed particles in micro/nanosensors with a flow-through reaction chamber can be analyzed. Also, as the variance of the random process $N(t)$ in the steady state (*i.e.*, the total mean power of the corresponding noise, called the adsorption-desorption noise) is:

$$\begin{aligned} \sigma_N^2 &= \int_0^\infty S_{\Delta N}(f) df = \frac{P^2}{4\tau} = k_f N_e \tau = \\ &= \frac{k_f k_r C N_m}{(k_r + k_f C)^2} \left(1 + \frac{k_f k_r N_m}{k_r + k_f C} \frac{1}{k_m A} \right) \end{aligned} \quad (8)$$

and its expected value N_e is given by Eq. (5). The presented theory can be used for the analysis of the influence of mass transfer on both the statistical parameters of the random process $N(t)$ and the noise caused by fluctuations of the number of adsorbed particles in adsorption-based micro/nanosensors. Apart from that, the analysis can be performed of the signal-to-(AD noise) ratio defined as the ratio of the signal total power and the total power of noise caused by stochastic AD processes coupled with mass transfer:

$$S/N = N_e^2 / \sigma_N^2 = N_e / (k_r \tau) \quad (9)$$

which is a significant indicator of the sensor's limiting performances.

RESULTS AND DISCUSSION

The presented theory is applied for the analysis of the influence of mass transfer processes on the plateau value of the fluctuations spectral density of the number of adsorbed particles (Figure 1) and on the signal-to-(AD noise) ratio (Figure 2) in microfluidic sensors. It is used in order to determine how pronounced the influence is at different target particles concentrations, different surface densities of functionalization sites, and different values of adsorption and desorption rate constants. The area of the functionalized surface is typical for microsensors, $A = 1 \times 10^{-9} \text{ m}^2$. In all of the given 3D diagrams in Figure 1, the lower surface is obtained by using Eq. (7) and assuming that the mass transfer is fast enough that it does not influence the kinetics of the process of binding the particles to the sensor's surface (this means that k_m is high enough that the second term in brackets in Eq.

(4) is $\ll 1$, thus $\tau = (k_r + k_r C)^{-1}$). It is intended for the comparison of the influence of each of the analyzed parameters on the effect the mass transfer has on the fluctuation spectrum. The upper surfaces of the 3D diagrams shown in Figure 2 also correspond to the case when mass transfer is neglected, and are given for the same purpose. They are obtained by using Eq. (9) with $\tau = (k_r + k_r C)^{-1}$.

All the diagrams in Figures 1 and 2 show that when the mass transfer is slow (*i.e.*, at lower k_m) the influence of the transfer on both the plateau magnitude and the signal-to-noise ratio (SNR) becomes significant: the plateau magnitude can be increased and the SNR decreased by more than one order of magnitude compared to the case of fast mass transport. Figures 1a and 2a show the dependences of the spectrum plateau and of the signal-to-noise ratio (SNR), respectively, on both the mass transfer coefficient and the target particles concentration at $k_r = 5 \times 10^7 \text{ 1/(M}\cdot\text{s)}$, $k_r = 0.08 \text{ 1/s}$ and $n_m = N_m/A = 1 \times 10^{-8} \text{ mol/m}^2$. At lower particle concentrations in the sample the influence of mass transfer on both the SNR and the plateau value becomes more pronounced. Figures 1b and 2b show the plateau magnitude and the

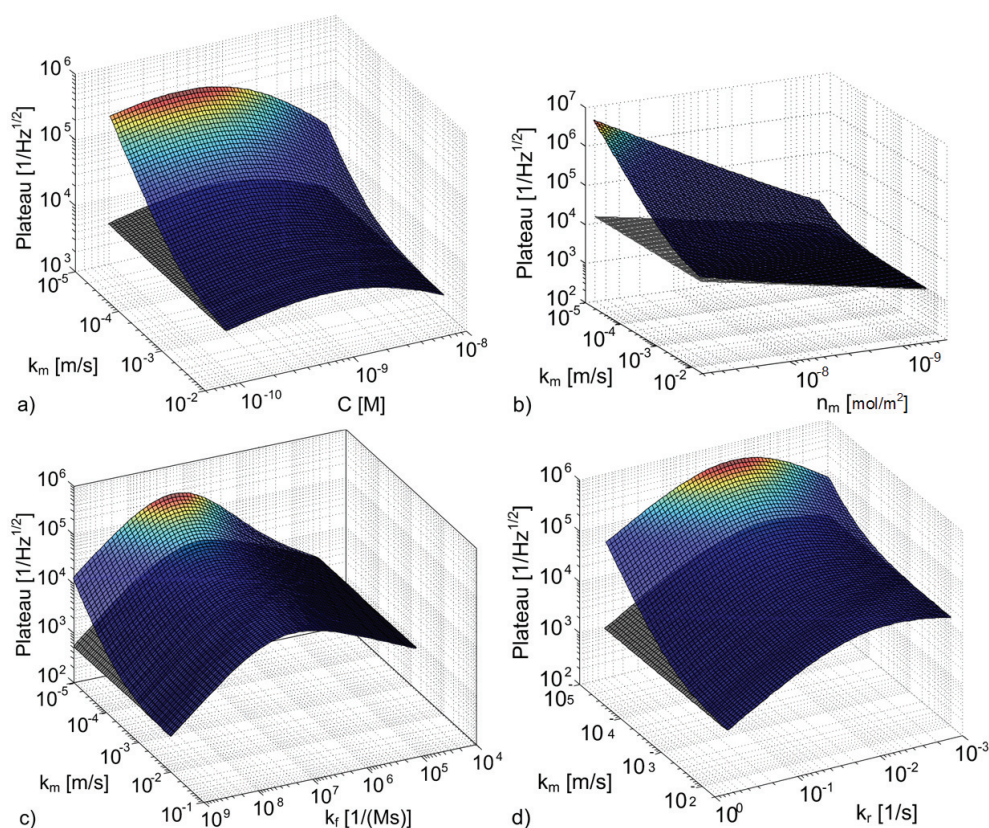


Figure 1. The influence of the mass transfer on the plateau value of the adsorbed particles number fluctuations spectrum for different: analyte concentrations (a), values of the functionalization sites surface density (b), adsorption rate constants (c) and desorption rate constants (d). The lower surfaces (shown in gray) are obtained by assuming that the mass transfer is fast enough that it does not influence the kinetics of the process of binding the particles to the sensor's surface.

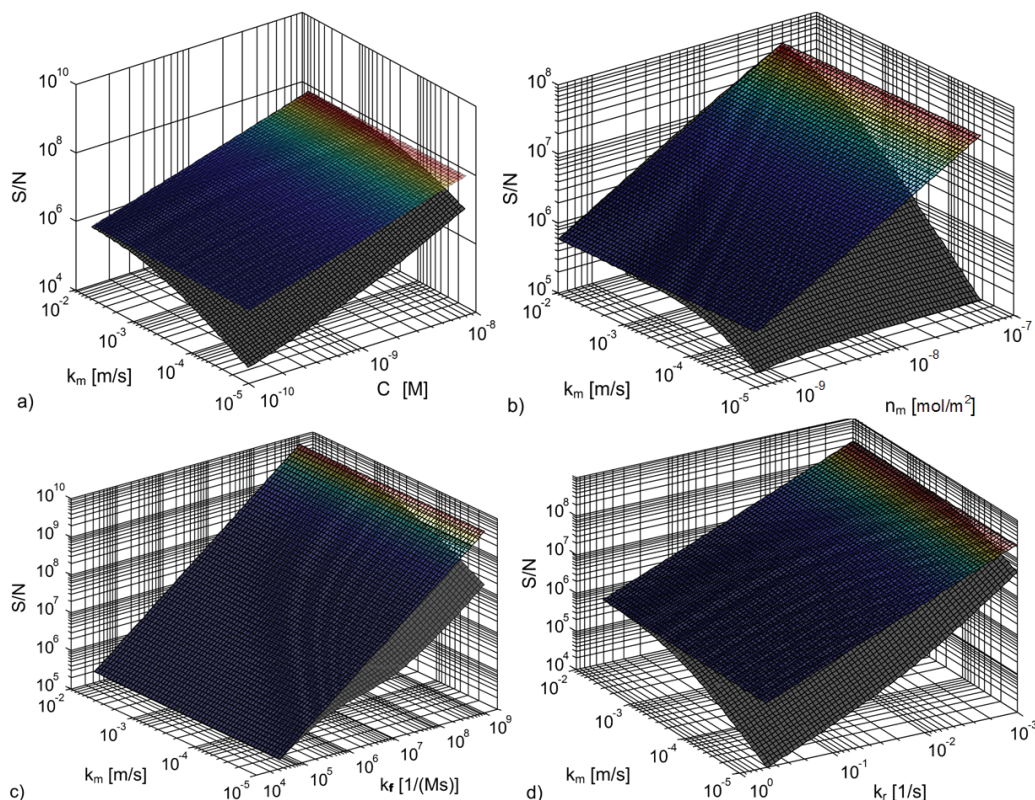


Figure 2. The influence of the mass transfer coefficient on the signal-to-(AD noise) ratio for different: analyte concentrations (a), values of the functionalization sites surface density (b), adsorption rate constants (c) and desorption rate constants (d). The upper surfaces correspond to the case when mass transfer is neglected.

SNR, respectively, as a function of both k_m and the surface density of adsorption sites on the sensor's surface ($k_r = 8 \times 10^7$ 1/(M·s), $k_r = 0.08$ 1/s, $C = 5 \times 10^{-10}$ M). In the case of slow mass transfer and high n_m , the plateau magnitude is more than 100 times greater than it is when the mass transport is fast. As the number of functionalized sites decreases so does the influence of the transport process speed on the plateau magnitude. The mass transfer influence on the decrease of SNR also becomes more pronounced with the increase of the surface density of adsorption sites. In Figures 1c and 2c the influence is shown of both the transfer process speed and the adsorption rate constant on the fluctuations spectrum plateau and on the SNR, respectively ($k_r = 0.02$ 1/s, $C = 2 \times 10^{-9}$ M, $n_m = 2.5 \times 10^{-8}$ mol/m²). At higher association affinities of the target particles for the sensing surface (*i.e.*, higher k_r), the mass transfer exhibits a higher influence on the fluctuation spectrum in the plateau range. Also, at higher k_r the influence of mass transfer on the SNR decrease is significant until k_m reaches higher values. In order for the analysis to be complete, it is also necessary to consider the dependence of the fluctuations spectrum plateau and of the SNR on both the k_m and the desorption rate constant (the

binding affinity constant is inversely proportional to k_r). These dependences are shown in Figures 1d and 2d, respectively, for $k_r = 5 \times 10^7$ 1/(M·s), $C = 2 \times 10^{-10}$ M and $n_m = 1 \times 10^{-8}$ mol/m². As it can be seen, the greater the k_r , the two surfaces on both diagrams become farther apart, thus the stronger the mass transfer influence on both analyzed parameters.

Based on the presented analysis it can be concluded that mass transfer adversely influences the ultimate sensor performances (the signal-to-noise ratio and consequentially the minimal detectable signal) and therefore its effects should be minimized. The derived analytical expressions can be used during sensor design and experimental conditions determination, as a tool for prediction and estimation of sensor performance (AD noise, signal-to-noise ratio, detection limit) for a given target analyte and functionalization layer. The expressions enable the optimal choice of sensor parameters. For example, by choosing the sample flow rate and the geometrical parameters of both the reaction chamber and the sensing surface, a certain increase of the mass transfer coefficient k_m for the given analyte can be achieved. Then, for the obtained k_m and based on the diagram shown in Figure 2b, one can estimate whether it is

possible to achieve the required signal-to-noise ratio by increasing the surface density of functionalizing sites and then can determine the optimal density value.

CONCLUSIONS

The theoretical model of fluctuations of the number of adsorbed micro/nanoparticles in environmental sensors operating in air and liquids is presented, taking into account the mass transfer processes of the target particles in a reaction chamber. Based on the expression for the fluctuations spectral density, the expressions for the total power of the corresponding adsorption-desorption noise and for the corresponding signal-to-noise ratio have been derived.

The performed analysis shows the significant influence of transport processes on both the plateau value of fluctuations spectrum and the signal-to-(AD noise) ratio, which are parameters that determine the sensor limiting performance. It also shows how significant this influence can be at different values of particles concentration, functionalization sites density, and adsorption and desorption rate constants. The results of the analysis provide the guidelines for optimization of sensor design and operating conditions (the flow rate of the sample through the reaction chamber, the functionalization sites surface density etc.) in order to decrease the influence of the transport processes, thus decreasing fluctuations and increasing the signal-to-noise ratio, *i.e.*, improving the ultimate performance of sensors for particle detection in the environment.

Nomenclature

AD - Adsorption-desorption

SNR - Signal-to-Noise ratio

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NAUČNI RAD

FLUKTUACIJE BROJA ADSORBOVANIH MIKRO/NANOČESTICA KOD SENZORA ZA MERENJE KONCENTRACIJE ČESTICA U VAZDUHU I TEČNIM SREDINAMA

Prikazan je teorijski model fluktuacija broja adsorbovanih mikro/nanočestica kod senzora za merenje parametara životne sredine, koji rade u vazduhu ili u tečnoj sredini. Model uzima u obzir uticaj procesa prenosa mase ciljnih čestica u reakcionoj komori senzora. Izvedeni su izrazi za snagu odgovarajućeg adsorpciono-desorpcionog šuma, kao i za odgovarajući odnos signal/šum. Prikazana analiza pokazuje da procesi prenosa mase mogu značajno da utiču na granične performanse senzora. Procenjen je uticaj prenosa mase na spektar fluktuacija i na odnos signal/šum pri različitim vrednostima koncentracije ciljnih čestica, površinske gustine funkcionalizujućih mesta i konstanti adsorpcije i desorpcije (izabrane vrednosti parametara su iz opsega koji odgovaraju realnim uslovima). Analiza omogućuje da se donesu zaključci potrebni za optimizaciju dizajna senzora i radnih uslova u smislu smanjenja uticaja prenosa mase, i da se time postigne poboljšanje graničnih performansi (npr. smanjenje minimalnog detektabilnog signala) senzora za detekciju čestica.

Ključne reči: senzori za merenje parametara životne sredine, mikro/nanočestice, adsorpcija, odnos signal/šum, prenos mase.