# Mihailo Petrović ALAS

## Life Work Times

Serbian Academy of Sciences and Arts

MIHAILO PETROVIĆ ALAS 150th birth anniversary

MIHAILO
PETROVIĆ
150th ALAS
birth anniversary

SERBIAN ACADEMY OF SCIENCES AND ARTS

MIHAILO PETROVIĆ ALAS: LIFE, WORK, TIMES
ON THE OCCASION OF THE 150th ANNIVERSARY OF HIS BIRTH

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# LIFE, WORK, TIMES

ON THE OCCASION OF THE 150$^{th}$ ANNIVERSARY
OF HIS BIRTH

SERBIAN ACADEMY OF SCIENCES AND ARTS

Exclusive editions, such as this monograph, call for the engagement, enthusiasm and cooperation of a number of individuals and institutions. We would like to use this opportunity and extend our gratitude to everyone who has taken part or in any way contributed to, or supported the creation and publication of this monograph.

First of all, we would like to express our gratitude to the authors of papers for their effort taken to provide expert and high level insights into some main points of Mihailo Petrović Alas' life and work, at the same time preserving an important aspect of being easy to read and appealing to a broader readership. In addition, we would like to thank to Ms. Snežana Krstić-Bukarica and Ms. Nevena Đurđević from SASA Publishing Section for performing a thorough proofread of the papers, thus making the writing even more articulate.

The monograph features a number of photographs and the copies of documents that have been obtained owing to the kindness of the SASA Archive, SASA Library, SASA Mathematical Institute, Archive of Serbia, Mr. Viktor Lazić from the "Adligat" Society, Mr. Jovan Hans Ivanović and his "Mihailo Petrović Alas" Foundation, "Mihailo Petrović Alas" Primary School, "Svetozar Marković" University Library, Belgrade City Museum, Zavod za udžbenike (Institute for Textbook Publishing) in Belgrade, Virtual Library of Faculty of Mathematics in Belgrade and Digital Legacy of Mihailo Petrović Alas.

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S. Pilipović, G. Milovanović, Ž. Mijajlović

# CONTENTS

## EDITOR'S FOREWORD

As soon as one first encounters the work of Mihailo Petrović, it becomes evident that he was a person that according to its numerous traits was a polymath. Above all, the academician Petrović was a gifted mathematician and a renowned professor at the University of Belgrade, but also a fisherman, writer, philosopher, musician, world traveler and a travel writer. He earned a degree in mathematics at the Belgrade Grand School and a licentiate degree in mathematics, physics and chemistry at the Sorbonne. At the age of 26, only a year after he had completed his studies, he defended his PhD degree in mathematics at the same university, as a student of the famous French mathematicians Henri Poincaré, Charles Hermite and Charles Émile Picard. In the same year (1894) he was elected to the position of professor at the Grand School to which he brought the spirit of the French mathematical school. It was at that point that his long and prolific journey through science began, whereas, owing to him, Belgrade achieved parity with other major European centers in mathematical sciences. He became an initiator and a leader of the Serbian mathematics and strongly contributed to the spirit of the modern European science in Serbia.

Petrović's expertize spanned several mathematical areas in which he achieved scientific results of world-class relevance: differential equations, numerical analysis, theory of functions of a complex variable and geometry of polynomials. He was also interested in natural sciences, chemistry, physics and biology, and he published scientific papers in these fields, too. In his scientific endeavor he managed to meet the most rigorous standards of the most developed European countries. In a brilliant rise, in a few years' time, up to the early 20$^{th}$ century, he wrote around thirty papers that he published in the leading European mathematical journals. It was due to this fact that he was elected a member of the Serbian Royal Academy as early as at the age of 30, and soon after he became a member of a number of foreign academies and prominent expert societies. He won the greatest respect of the global mathematical community: he was among few mathematicians (13) who delivered at least five plenary lectures or lectures as a visiting lecturer at the International Congress of Mathematicians (ICM). He delivered five such lectures (1908, 1912, 1924, 1928 and 1932). One such invitation has been considered by the mathematical community as an equivalent of an induction to a hall of fame. In addition, it has been considered that Petrović was a founder of new scientific disciplines, namely mathematical phenomenology and spectral theory. He invented several analogue computing machines, possessed technical patents and was the main cryptographer of the Serbian and Yugoslav Army.

Up to the Second World War he was the mentor of all doctoral thesis in mathematics defended at the University of Belgrade. Aforementioned is related to one of professor Petrović's greatest and most important achievements – he was a founder of the Serbian mathematical school that has produced a great number of renowned and successful mathematicians not only in Serbia but also around the world.

In 2018, the Serbian Academy of Sciences and Arts and mathematicians in Serbia celebrate the 150th anniversary of the birth of Mihailo Petrović Alas. Throughout this year, the Academy has organized a large exhibition dedicated to Petrović, alongside a solemn gathering and a conference. This monograph commemorates this important jubilee of the Serbian mathematics. Given the fact that a lot of articles on Petrović have already been written, and that his collected works were published at the end of the last century, the editors and authors of the papers in this monograph were faced with a daunting task of finding some new details from professor Petrović's life and career. Even more so given that his body of work is immense, spanning different scientific areas and encompassing topics that at first glance one finds difficult to combine. As Dragan Trifunović, Petrović's biographer and a man who most thoroughly studied his life and work, noted on one occasion that almost an institute was necessary that would encompass professor's entire body of work. Therefore, we set a relatively modest goal to ourselves to shed light upon some main points of Petrović's life and work, times and circumstances he lived in, as well as to elaborate on the present developments in relation to the Serbian mathematical school, through a selection of papers. The authors of the papers steered clear of technical details and excessive use of mathematical language. Hence, the monograph is intended for a broader readership, in particular to those readers who are interested in the history of Serbian science and its evolvement at the turn of the 20th century, but also to those who want to gain a deeper insight into the life of a brilliant mathematician and a polymath, and, we can quite freely say, an unusual personality.

Ž. Mijajlović, S. Pilipović, G. Milovanović

# MIHAILO PETROVIĆ ALAS: LIFE AND WORK

# MATHEMATICAL PHENOMENOLOGY BETWEEN MYTH AND REALITY

Nikola PETROVIĆ MORENA
*Morena inženjering, Niš*

Mathematical phenomenology is a relatively well-known term in Serbia owing to the works of Mihailo Petrović Alas. Even though none of Petrović's students or the other Serbian scientists continued the work in that field (or perhaps for that very reason), it acquired a certain mystical aura. At times, one could read (or, even more likely, hear) an opinion that Petrović's phenomenology has yet to be properly interpreted. However, searching the Internet for the term "mathematical phenomenology" yields just a few results, apart from the translations of Serbian papers, and contemporary encyclopaedias, for the most part, do not contain an entry referring to that topic. Does mathematical phenomenology exist at all or is it a part of national mythology nurturing the stories about the unrecognized grandeur of our scientists? If it exists, what is mathematical phenomenology concerned with and how can it be distinguished from mathematical modelling which, at least at first glance, seems very similar to it? And finally, what is the unique contribution that Petrović made to that field? This investigation represents an attempt to give some of the possible answers to those questions.

## PHENOMENOLOGY AS A PHILOSOPHICAL CONCEPT

Immanuel Kant (1724–1804)

The word "phenomenology" is derived from the Greek words *phainómenon* ("appearance") and *lógos* ("study, research"), so one of the meanings of this term is "the study of phenomena". If we translate *lógos* differently in this compound word,[101] we will get alternative meanings of phenomenology – "the appearance (revelation) of the first principle, the all-encompassing law or spirit".

In *The Critique of Pure Reason* (1781), Immanuel Kant pointed out the difference between the "phenomena", man's interpretation of an object or an event based on the information apprehended by the senses, by reasoning or through experience, and the "noumenon", an object or an event in itself, which is inaccessible to man. According to Kant, human reason is actively involved in acquiring knowledge about the world, trying to put the phenomena into "matrices" that are already present in his consciousness. The main matrices exist in man a priori, that is, prior to experience.[102] Such a priori matrices include space and time. Science deals with the world of phenomena, with the apprehensible, and theology with the unknowable, the noumenon.[103] Kant has proved that striving towards the metaphysical truths of the noumenal world through reason invariably ends up in contradiction.

Hegel accepts Kant's distinction between phenomenon and noumenon, but denies his claim that the essence of things is not apprehensible by man. In *The Phenomenology of Spirit* (1807) and other works, Hegel develops an idea of phenomenology as a philosophical method which starts with what can be apprehended by the consciousness – phenomena, and by deepening the knowledge about the phenomena, ends in reaching the absolute, metaphysical spirit – logos, which lies behind phenomena.

By the end of the nineteenth and the beginning of the twentieth century, positivism had become a dominant trend in philosophy. Positivism aims to deal only with the things that are "positive", verifiable, and it banishes metaphysics from philosophy.[104] Within the positivist approach, phenomenology has been equated with the scientific view according to which it is vital that the phenomena of the so-called empirical world be described and anticipated as accurately as possible, without ever questioning their purpose.

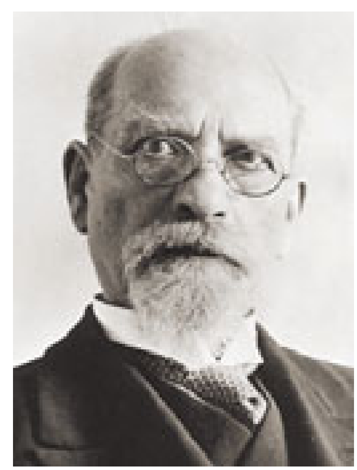
In the 1920's, Edmund Husserl, a German mathematician and philosopher of Jewish origin, laid down the foundations of phenomenology as an independent philosophical movement and made the term become more widely used. Husserl makes a departure from positivism, asserting that there exists a spiritual reality independent of the material world and that studying that reality is to be the basic aim of science, which has "strayed", especially in Europe, focusing only on the empirical and natural. The essence of things and phenomena (Husserl uses the term "essences") exist in our consciousness and we can grasp them by gradually discarding all that is variable in the phenomena. That process is called the *phenomenological reduction*. According to Husserl, phenomenology is a method of philosophical investigation which requires that the investigator eliminate all preconceptions and assumptions,[105] which enables him to look at things with an open mind and to understand their meaning through the interaction between his own consciousness and the observed object [Moustakas 1994].

Natural sciences are founded upon axiomatically accepted paradigms and hypotheses. An astronomer assumes the physics to be accurate and a physicist relies on the truthfulness of mathematics in the same way as a mathematician relies on logic. The advantage of a phenomenological method lies in the fact that the analysis begins with lived experience and does not require any a priori assumptions whose validity lies beyond the domain of a concrete investigation. Owing to that, a phenomenological investigation has got a fundamental character. One of the axiomatic assumptions of the natural sciences is the existence of reality that lies outside of man's consciousness and is independent of it. A phenomenologist rejects such an assumption. He does not deny the possibility that such a reality exists, he does not even doubt it, he simply refrains from passing judgement on that matter. A phenomenologist is trying to explain the world strictly by analyzing the experiences within his own consciousness through systematic reflection. Phenomenology is trying to build a framework for an objective, scientific studying of topics that are usually considered subjective, such as consciousness, reasoning, perception, or emotions, but using methods different from those applied in clinical psychology or neurology.

Unlike analytical philosophy, which is mostly concerned with analysis of utterances and sentences, phenomenology deals with experiences and their structure [Пивчевић 1997]. The structure of a linguistic utterance, as phenomenologists claim, cannot really be understood without analyzing the structure of experiences that lend meaning to those utterances.

The goal of phenomenology lies not in new empirical knowledge, but in understanding our fundamental relation to the world which

Georg Wilhelm Friedrich Hegel (1770–1831)



Edmund Husserl (1859–1938)

Martin Heidegger
(1889–1976)

Jean-Paul Sartre
(1905–1980)

Maurice Merleau-Ponty
(1908–1961)

comes before any empirical investigation [Zahavi 2008: 664–665]. It endeavours to describe rather than analyze, and that is the *fundamental phenomenological instruction* [Merleau-Ponty 1990]. Thus, in the Husserlian context, phenomenology acquired a markedly different meaning from the one used within positivism.

A similar view is held by a German psychotherapist Bert Hellinger, according to whom there are two ways towards an insight. One traverses the unknown, revealing to our reason the secrets of the world around us, step by step. That is the way of science. The other way requires of us to stop our trying to understand things and, in turn, let our attention become ever broader in scope, ever more expansive, until it is able to contain the whole instead of the parts. Such renunciation of analysis and giving priority to sensory experience represents the foundation of the phenomenological method.

The further development of phenomenology during the twentieth century has brought new, often critical interpretations of Husserl's views and new philosophical trends, including the existentialism. The most prominent philosophers of the twentieth century who based their doctrines on phenomenology were Martin Heidegger, Jean-Paul Sartre and Maurice Merleau-Ponty.

## PHENOMENOLOGY AS A SCIENTIFIC METHOD

In science, phenomenology is a method which comes to the essence of a matter (noumenon) by using written accounts of lived experience (phenomena) as sources of knowledge [Conklin 2007: 275]. Phenomenological method is used in statistical sociological research, when the members of a target group in a research study are asked to respond to the questions presented in forms. Their responses are then coded in numerical values which are amenable to a mathematical treatment, mostly with a view to establishing an average value.

A scientific theory which mathematically expresses the findings of an investigated phenomenon, without considering the essence of the phenomenon (noumenon), which lies behind it, is called a phenomenological theory [Thewlis 1993: 248].

There are scientific subjects, such as astronomy, in which there is a very limited scope for conducting experiments. It is very hard to analyze the appearance and disappearance of a star in laboratory conditions [Божић 2005: 24]. In subjects of this kind, studying the phenomena is the basic scientific method.

## PYTHAGORAS AS A PRECURSOR OF MATHEMATICAL PHENOMENOLOGY

If phenomenology is understood to be a process of reducing the phenomena to their "essences", and mathematical phenomenology is viewed as the kind of phenomenology that finds the said essences in numbers and their relations, then Pythagoras can be said to represent a precursor of mathematical phenomenology. He thought that the world had a mathematical character at its root. Studying music, he spotted the relation between the length of a string on a lyre, the most popular musical instrument in his time, and the frequency of the tone the string produces when it oscillates. Observing astronomical phenomena, the orbits of the planets, the length of days and nights, he found everything to be numerically related. That all things are number was the main Pythagorean dogma [Божић 2010: 53].

Moreover, Pythagoras and his disciples, the Pythagoreans,[106] had not only thought the world could be described in terms of numerical relations, but also that numbers as well as nature are governed by the same principle: both the infinite series of "natural numbers" and the whole universe represent the relation between the finite and the infinite – "peiron" and "apeiron". The Pythagoreans have taken over the terms peiron and apeiron from Anaximander, and they complemented his teachings with the thesis that the relation between apeiron and peiron is ruled by the principle of harmony. For instance, the infinite series of musical tones (apeiron) has to be limited in some way (peiron) in order to form a scale. However, we cannot choose any series of tones so as to produce a scale that is to be musically agreeable, harmonious. Only the tones whose frequencies are related by whole-number ratios sound harmonious! In the same manner, the universe and all living creatures in it are not created by a random combination of finite and infinite elements, but by elements combined in a harmonious way in numerical proportions, which all together constitute the cosmic order.

What the Pythagoreans meant by a number was what we now call positive rational numbers – natural numbers and their relations. When they had discovered that in nature there are values that cannot be represented by rational numbers, such as the diagonal of a square whose side is a natural number, their view of the world was shaken up to its foundations. That discovery was kept as a great secret of the inner circle of Pythagoras's followers. It is said that certain Philon divulged the secret, and was forced to commit suicide by jumping from a high cliff into the sea.

Pythagoras

In Pythagoras's reduction of everything to a number, we can sense the method that much later Husserl will come to call the phenomenological reduction. Pythagoras was not interested in the empirical (or "scientific", as we would now say) approach to phenomena, he was only trying to describe them using a mathematical model without entering into the analysis of the causes of a phenomenon. This is especially evident in Pythagoras's astronomical investigations. *The emancipation of mathematics from empiricism*" [Божић 2010: 126], which had been conducted by the Pythagoreans, enabled mathematics to start developing as an independent discipline.

# THE PHENOMENOLOGY OF "MATHEMATICAL PHENOMENOLOGY"

Even though the ideas upon which mathematical phenomenology is based had appeared much earlier, the term itself started to be used in the late nineteenth century in parallel with the development of the positivist school of philosophy.

The claim that mathematics is the only solid epistemological paradigm and that every scientific finding has to be based on it (or even reducible to mathematics) was made among the first by a French philosopher René Descartes in his work *Discourse on the Method of Rightly Conducting One's Reason and of Seeking Truth in the Sciences* of 1637 [Божић 2010: 183]. A very influential work of Isaac Newton, *Mathematical Principles of Natural Philosophy*[107] of 1687, contributed to spreading the idea that all exact sciences have to be amenable to a mathematical formulation.

Among world-renowned scientists in the late nineteenth century, the term mathematical phenomenology was used by an Austrian physicist Ludwig Boltzmann, German physicist Gustav Kirchhoff, after whom the laws on the conservation of the quantity of charge in closed electrical circuits were named, as well as Heinrich Hertz, a German scientist after whom a unit of frequency was named [Hon and Rakover 2001: 9]. In the introduction to the book *The Principles of Mechanics,* which was posthumously published in 1894, Hertz wrote that physicists must focus on finding equations by which they can determine the development of phenomena in quantitative terms, without using any hypotheses, non-mathematical models or mechanical explanations. According to Hertz, Maxwell's theory is a classical example of such an approach. To the question – "What is Maxwell's theory?" – there is no shorter or more accurate answer than the following: "It is Maxwell's system of equations", wrote Hertz. Such an approach was very well-known to the positivists, who were on the rise at the time. Their ideal was not only philosophy freed from metaphysics, but also physics (and science in general) freed from "mythology", that is, from metaphysical systems that are trying to make sense of the world. The question "why" had been banished from science, and the pivotal place was taken over by "how much" and "in what manner". The majority of positivists later softened their original views, presenting them as a reaction to the historical moment in which philosophy had been dominated by German idealism.


René Descartes (1596–1650)

In the article on models for the tenth edition of *Encyclopaedia Britannica* of 1902, Ludwig Boltzmann explained that mathematical phenomenology represents a specific view on the nature of physical theories according to which the goal of a physical theory should primarily be the construction of mathematical formulae by which the observed phenomenon can be quantitatively described in a way that is closest to reality. Boltzmann named Kirchhoff and his school as typical representatives of mathematical phenomenology. "Mathematical phenomenology is a presentation of phenomena using mathematical analogies", wrote Boltzmann (1902).

Boltzmann also wrote about a radical variant of a mathematical phenomenology framework, according to which the equations describing a phenomenon are more important (or, at least, more purposeful) than an attempt to discover the cause of a phenomenon [Feuer 1989: 337]. According to that viewpoint, the hypotheses-paradigms based on which the equations are formed are not permanent and they change with the development of science, but empirically conducted and verified mathematical formulae describing physical phenomena remain valid even after a paradigm shift, except possibly in marginal scopes of measured values having lied outside of the domain of empiricism when the formulae were being created [Boltzmann 1901: 248–250]. Boltzmann has criticized the phenomenological approach by stating that it is impossible to understand nature by relying solely on the empirical. He has especially criticized mathematical phenomenology, claiming that no set of equations can ever give a complete description of a phenomenon.

Since the end of the first decade of the twentieth century, apart from the papers related to Mihailo Petrović Alas, very few references in literature related to the term "mathematical phenomenology" have been published. The most comprehensive world encyclopaedias (even those specializing in mathematics) do not contain entries regarding that field. According to data available on the Internet, the only higher education institution holding a course in mathematical phenomenology is "Waseda School of Science and Engineering" in Japan. However, it should be pointed out that even to this day there are many scientists, magazines and scientific conferences dealing with the relation between mathematics and phenomenology.

## THE DIFFERENCE BETWEEN MATHEMATICAL PHENOMENOLOGY AND MATHEMATICAL MODELLING

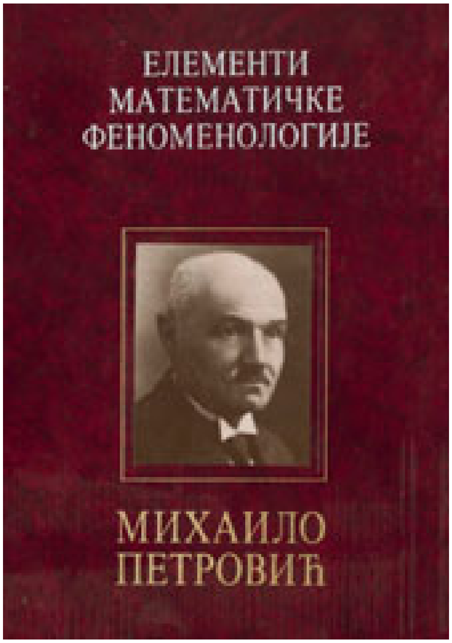
Unlike mathematical phenomenology, the term mathematical modelling is in widespread use. There are thousands of books and university courses dealing with mathematical modelling.

A mathematical model is a description of a system by using mathematical language and concepts. The process of creating a mathematical model is called mathematical modelling. The difference between this definition and Boltzmann's definition of mathematical phenomenology (the presentation of phenomena by using mathematical analogies) is a subtle one and lies at the level of linguistic and philosophical preferences.

It might be said that mathematical modelling is a practical skill within the domain of applied mathematics, whereas referring to mathematical phenomenology usually entails a philosophical viewpoint or at least a philosophical background. This distinction is not sharply defined, as Boltzmann himself [1901: 250] pointed out that mathematical phenomenology has got a primarily practical purpose. The author of this paper is of an opinion that between mathematical phenomenology taken in positivist context as defined by Boltzmann on the one hand, and mathematical modelling on the other, there is no essential difference. In Boltzmann's time, mathematicians were almost invariably philosophers, as well. However, the twentieth century saw many generations of mathematicians-craftspeople, and the term mathematical phenomenology was accordingly superseded by an intellectually less challenging term mathematical modelling.

Boltzmann's definition of mathematical phenomenology is not the only one. Some contemporary authors, such as professor Doorman from Utrecht University, think that *mathematical phenomenology refers to how mathematical ideas structure and organise phenomena* [Doorman 2005: 59]. Thus defined, mathematical phenomenology represents, in fact, the philosophy of mathematical modelling.

Ludwig Boltzmann (1844–1906)




The cover page of *Elements of Mathematical Phenomenology*, *Collected Works*, Book 7 (Digital Legacy of Mihailo Petrović)

## MATHEMATICAL PHENOMENOLOGY OF MIHAILO PETROVIĆ ALAS

Mihailo Petrović Alas (1868–1943) is a renowned Serbian mathematician, physicist, travel writer, violinist and a fisherman. As a government scholarship holder, he obtained his doctorate in mathematics and physics in Paris. He spent his whole scientific career working at the University of Belgrade. The greatest impact on the community of experts was made by his papers in the field of differential equations. He is one of the first Serbian scientists whose works have been cited in Europe. His interests were very versatile and reinforced by his encyclopaedic knowledge [Трифуновић 1998: 366]. He travelled as a member of scientific expeditions to the North and South Pole. His virtuosity on the violin was recorded on one of the earliest audio files of that instrument produced by Radio Belgrade [Трифуновић 1991: 15]. It could be said that he was a polymath, homo universalis[108], one of the rare Renaissance people in modern Serbia (Божић 2005). Mihailo Petrović Alas had friends among people from diverse social backgrounds and in Belgrade downtown community he has been remembered as "our Mika".

Out of the fifteen volumes of his collected works published by "Zavod za udžbenike i nastavna sredstva", two volumes comprising about 1000 pages in total are devoted to mathematical phenomenology. They include twenty published Petrović's works (books, papers, speeches...) on that topic and list more than one hundred references in which other authors commented on Petrović's phenomenology. The editor entitled those volumes *Mathematical Phenomenology* and *The Elements of Mathematical Phenomenology*.

At the heart of Petrović's interest in those works lies the concept of phenomenological mapping. He realized that the phenomena from different areas of human experience (Petrović's term: "disparate phenomena") can be reduced to, that is, mapped onto the same abstract essence (Petrović's term: "a phenomenological type of facts") [Петровић 1998a: 13]. For instance, the phenomena of the height of tsunami waves being reduced as their distance from the place of origin increases, the waning of military power of a conqueror in the face of vast expanses, a reduction in the intensity of light in proportion to its distance from the light source, all of which represent disparate phenomena that belong to a common phenomenological type – weakening in proportion to expanding.

The turning of the tide or of the day into night have the same phenomenological type as some phenomena disparate in relation to those, like the menstrual cycle – periodical changes induced by a periodical cause. There is an obvious analogy with Husserl's terminology: phenomenological mapping refers to phenomenological reduction, and phenomenological types refer to essences. The roles (Petrović's term: "phenomenological beings") contained in the phenomenological type of facts are independent of the concrete nature of the holder of the role. In our first example, tsunami, military charge and light wave have the role of an impulsive factor, whereas the ocean, the expanse of a state that is being conquered as well as the space through which the light wave is spreading have a territorial role.

What is the goal of phenomenological mapping according to Petrović? The goal is to step a little closer to the ideal, ultimate goal of "positive philosophy", the reduction of an infinitely colourful view of the world to the most simplified sketch that underlies it, but such that the original picture could be reproduced from it by adding specific, phenomenologically insignificant details that do not contradict the sketch [Петровић 1998a: 17]. The significance of phenomenological mapping lies in the possibility of predicting the details in the phenomena whose phenomenological type we have identified even if we do not understand their essence. Those details are not exclusively related to the number and for that reason Petrović proposed founding a new subject whose methods would include all the details that could be completely abstracted from concrete phenomena and studied in themselves, as it is done in mathematics with abstracting numbers [1998a: 18]. Petrović defined that subject as "a new branch of the philosophy of nature that would comprise general methods for predicting phenomena based on the nature of the roles of the factors that represent the cause of a phenomenon" [1998b: 14]. What is the name of that new branch of the philosophy of nature?

A more attentive reader would be perplexed, even upon browsing through the volume entitled *Mathematical Phenomenology* – none of the five Petrović's works published in it contain in their title, or even in the chapter headings, the term mathematical phenomenology. The editor of these collected works mentions this illogicality himself in the afterword and he justifies it by saying he did not know how to name the field with which Petrović had dealt with in these works and that he gave that title to the volume "in order to arrive at a natural and necessary unity" [Трифуновић 1998: 420] with the title of the following volume, in which the central place is held by Petrović's work *The Elements of Mathematical Phenomenology.*

And indeed, it had not been an easy task for the editor. Although Petrović has kept the said definition of the subject of his research for about forty years of working in that field, the terms he used for that field varied a lot. In his inaugural address on the occasion of being elected a full member of the Serbian Royal Academy in 1900, Petrović named it "a mathematical theory of actionality", only to change the title into "a mathematical theory of the activity of causes" for the print version of the very same speech [1998c: 222]. Five years later, in the debate *An Attempt at a General Mechanics of Cause,* the field became "the general mechanics of cause" [Трифуновић 1998: 274], and in 1911 in *The Elements of Mathematical Phenomenology,* Petrović named it "mathematical phenomenology". However, he soon relinquished even that appellation

Branislav Petronijević
(1875–1954)

and in the book *Analogies as a Basis of General Phenomenology,* of 1922, he uses the term "general phenomenology". In his philosophically most complete work, *Phenomenological Mapping,* of 1933, Petrović is very careful to avoid using the term "mathematical phenomenology" and in general naming the field of his research, except in one place where he called it "mathematics in an extended sense" [Петровић 1998a: 18].

In spite of the proverbial saying "you can call a pot a jug if you will, as long as you don't break it", it is very likely that Petrović's undecidedness about naming that field was one of the reasons for the fact that his work did not have a wider reception. One has to admit, it is very difficult to popularize a field which even its pioneer cannot name. It is evident that mathematical phenomenology was only one of the terms Petrović had used and later discarded. Moreover, it is clear that Boltzmann's encyclopaedic definition of mathematical phenomenology (a presentation of phenomena using mathematical methods) refers only to a subset of Petrović's mathematical phenomenology as a new branch of the philosophy of nature, which comprises general methods for predicting phenomena based on the nature of the roles of the factors that represent the cause of a phenomenon. It seems reasonable to assume that Petrović had realized that the term mathematical phenomenology was already burdened with the other, narrower meaning, so he decided not to use it in order to avoid causing confusion.

If we accept the "fundamental phenomenological instruction" and pay attention to Petrović's descriptions instead of the terms he used, it is clear that it is philosophy based on phenomenological concepts. Which area of philosophy is this?

In *Phenomenological Mapping* Petrović wrote that phenomenological types of facts and phenomenological beings cannot be discovered within any single scientific field because, irrespective of how broad a field is, it is always concerned with one concrete nature of phenomena.

In order to reach the essence, “in our mind we should erase the boundaries between certain fields and view the world directly, the world in which one and the same details thread through the infinite colourfulness of external phenomena, their outer guise” [Петровић 1998а: 12]. Only then is it possible that, by direct observation, scientific analysis or poetic intuition, we can manage to abstract a common core out of the myriad of disparate phenomena. Therefore, Petrović’s method requires a return to the original philosophy, the first philosophy, such as it had been before specific sciences were derived from it. In spite of the fact that Petrović frequently refers to a “positive philosophy”, his science is the science of being – metaphysics! That may explain why he failed to “circumscribe” the field of his research by some term, and put it within the framework of mechanics, classical mathematics or any other specific scientific field.

The Swiss psychiatrist C. G. Jung would call it an instance of synchronicity that almost at the same time when Petrović was writing *Phenomenological Mapping* in Belgrade, only several hundred kilometres to the north, in the Hungarian plains, Béla Hamvas began his essay *Poetica Metaphysica*[109] with the following words:

> “There were times and there are peoples whose religion, science, philosophy and poetry were all one, and are one even now... However, at times like these, reality has different areas, layers, planes, and they are segmented from one another... The one who crosses from one field into another is looked upon as though he has made a false step. The one who breaks barriers, bring areas together, joins the planes, is simply said to have become insane.”

Was it the very fear of the reaction of the public and the loss of reputation of a positivist mathematician that prevented Petrović (and consequently the editor of his collected works) from defining this area more clearly as deeply philosophical? Or perhaps he did it out of consideration for his colleague from the University, the philosopher Branislav Petronijević, who had conceived as his life’s work precisely what Petrović managed to do to a much greater extent – to put metaphysics on the firm ground of logic, notwithstanding the prevalent view holding that even Kant in his time had proved it to be impossible?[110]

The fact that Petrović has made a solid philosophical system is also demonstrated by his original ontology. In his phenomenological works, Petrović has introduced over one hundred terms that are either completely new or he has given them a quite different meaning [Трифуновић 1998: 416–420].

Petrović's universality is also reflected in the fact that he was one of the rare philosophers who were both metaphysicists and practitioners. The book *Phenomenological Mapping* offers on hundreds of pages not only detailed instructions for inductive abstraction of phenomenological types of facts from versatile phenomena, but also quicker, deductive methods for their identification, followed by the methods for predicting future phenomena solely based on thus abstracted types, as well as the methods for inverse phenomenological mapping, which enables an abstract phenomenological type of facts to be mapped onto an insufficiently known concrete area (of nature, psychology, economy, etc.) and predict phenomena within it. For those who deal with artificial intelligence, a matter of particular interest is Petrović's vision of a general phenomenology as a tool which, once it is sufficiently developed, "will have that power to think for us and yield results surpassing human reasoning" [1998a: 20]. The other Petrović's seminal work from this area, *The Elements of Mathematical Phenomenology*, focuses on those types of analogies among disparate phenomena that can be expressed by the existing, classical mathematical apparatus, particularly by differential equations. With regard to that matter, Petrović is completely within the area of his specialized expertise, and extensive knowledge of mathematics is required in order to fully understand Petrović's accomplishments in that area.

Many ideas of Petrović, (probably) independently of him, have been further developed in such diverse areas as cybernetics, psychology, economics or mythology. For instance, Petrović stated that concrete myths of various peoples are frequently very similar, because their essence is represented by the same phenomenological type of facts [1998a: 197–208]. Jung has arrived at the same conclusions, only using different terminology – Jung's archetypes of collective unconscious are completely compatible with Petrović's phenomenological type of facts.[111] Petrović has gone even further than Jung, seeing in paradigms of contemporary science "scientific mythology", which is only another expression of the same phenomenological type of facts that is to be found in classical mythology. What is the modern scientific entity of "force" which pulls, pushes, attracts, but a manifestation of the same phenomenological being that also represents the essence of mythological Eros, concluded Petrović [1998a: 199].

Transactional analysis of a Canadian psychiatrist Eric Berne (whose most famous book –*The Games People Play* has been translated into Serbian as *Koju igru igraš?*) shows that social interaction functions in terms of small variations of a limited set of scenarios and roles. The said concepts are completely analogous with Petrović's phenomenological roles and types.

In investigations of "small-world networks" it has been noticed that many networks, like social networks (e.g. Facebook), neural networks in the brain or the Internet, have an unusual feature in that an average distance between two randomly chosen knots (a person, neuron, computer), measured by the number of knots between them, is much shorter than it could be expected considering the size of the network and that it is proportional to the logarithm of the total number of knots in the network.[112] That is only one illustration of what Petrović used to call mathematical analogies in disparate facts [1998a: 71].

An advantage of Petrović's general phenomenology in relation to the above-mentioned examples lies in its universality – while Jung's and Berne's investigations refer to isolated areas of human experience, and small-world networks look at one type of analogies among disparate phenomena, Petrović's phenomenology includes all the areas as well as all the types of analogies.

The term "mathematical phenomenology", as well as the word phenomenology itself, is polysemous. In the most frequent, positivist context, mathematical phenomenology denotes a presentation of phenomena by using mathematical analogies, which is very close to mathematical modelling. In another context, it refers to investigating the ways in which mathematical ideas structure and categorize phenomena, in which case it is associated with the philosophy of mathematical modelling. Mihailo Petrović Alas founded a new branch of the philosophy of nature, which comprises general methods for predicting phenomena based on the nature of the roles of the factors representing the cause of a phenomenon. That field of his research was at one period named mathematical phenomenology, but later he relinquished that term with reason, because a presentation of phenomena by using mathematical analogies represents only one component of his philosophy. Petrović's original contribution to phenomenology lies in developing universal, practically applicable methods of phenomenological reduction and inverse phenomenological mapping, which could also readily be applied in contemporary artificial intelligence.

## REFERENCES


Boltzmann, L. E. (1901). "The Recent Development of Method in Theoretical Physics". *The Monist*, Vol. 11 (No. 2).

Boltzmann, L. E. (1902). "Model". In: *Encyclopedia Britannica*. London. Retrieved from http://www.muellerscience.com/MODELL/Definitionen/Encyclopaedia_Britannica.htm

Божић, М. (2005). „Математичка феноменологија Михаила Петровића". У: *Леīенде Беоīрадскоī универзиѿеѿа* (23–27). Београд: Универзитетска библиотека Светозар Марковић.

Божић, М. (2010). *Преīлед исѿорије и филозофије маѿемаѿике*. Београд: Завод за уџбенике.

Conklin, T. (2007). "Method or Madness: Phenomenology as Knowledge Creator". *Journal of Management Inquiry*, 16.

Doorman, L. M. (2005). *Modelling motion: from trace graphs to instantaneous change.* Utrecht: Dissertation Utrecht University. Retrieved from http://igitur-archive.library.uu.nl/dissertations/2005–0311-094207/full.pdf

Feuer, L. S. (1989). *Einstein and the Generations of Science*. New Jersey: Transaction Publishers.

Hon, G. and S. S. Rakover (eds.) (2001). *Explanation: Theoretical Approaches and Applications (Synthese Library)*. Berlin: Springer.

Hartimo, M. (2010). *Phenomenology and Mathematics*. Berlin: Springer.

Merleau-Ponty, M. (1990). *Fenomenologija percepcije*. Sarajevo: Veselin Masleša/Svjetlost.

Moustakas, C. (1994). *Phenomenological Research Methods*. New York: SAGE Publications.

Петровић, М. (1998a). *Феноменолошко ӣресликавање*. Сабрана дела, 6. том. Београд: Завод за уџбенике и наставна средства.

Петровић, М. (1998b). *Елементи математичке феноменологије*. Сабрана дела, 7. том. Београд: Завод за уџбенике и наставна средства.
Петровић, М. (1998c). *О математичкој теорији активности узрока*. Сабрана дела, 6. том Београд: Завод за уџбенике и наставна средства.
Pivčević, E. (1997). *Na tragu fenomenologije*. Zagreb: Globus.
Poper, K. (2002). *Pretpostavke i pobijanja*. Sremski Karlovci: Izdavačka knjižarnica Zorana Stojanovića.
Thewlis, J. (1973). *Concise Dictionary of Physics*. Oxford: Pergamon Press.
Трифуновић, Д. (1991). *Бард српске математике Михаило Петовић Алас: прилог интелектуалној биографији.* Београд: Завод за уџбенике и наставна средства.
Трифуновић, Д. (прир.) (1998). *Сабрана дела Михаила Петровића Аласа,* 6. том. Београд: Завод за уџбенике и наставна средства.
Zahavi, D. (2008). "Phenomenology". In: *Routledge Companion to Twentieth-Century Philosophy*. London: Routledge.