Mihailo Petrović ALAS

Life Work Times

Serbian Academy of Sciences and Arts

MIHAILO PETROVIĆ ALAS 150th birth anniversary

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150th birth anniversary

SERBIAN ACADEMY OF SCIENCES AND ARTS

MIHAILO PETROVIĆ ALAS: LIFE, WORK, TIMES
ON THE OCCASION OF THE 150th ANNIVERSARY OF HIS BIRTH

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MIHAILO PETROVIĆ ALAS
LIFE, WORK, TIMES

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OF HIS BIRTH

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Exclusive editions, such as this monograph, call for the engagement, enthusiasm and cooperation of a number of individuals and institutions. We would like to use this opportunity and extend our gratitude to everyone who has taken part or in any way contributed to, or supported the creation and publication of this monograph.

First of all, we would like to express our gratitude to the authors of papers for their effort taken to provide expert and high level insights into some main points of Mihailo Petrović Alas' life and work, at the same time preserving an important aspect of being easy to read and appealing to a broader readership. In addition, we would like to thank to Ms. Snežana Krstić-Bukarica and Ms. Nevena Đurđević from SASA Publishing Section for performing a thorough proofread of the papers, thus making the writing even more articulate.

The monograph features a number of photographs and the copies of documents that have been obtained owing to the kindness of the SASA Archive, SASA Library, SASA Mathematical Institute, Archive of Serbia, Mr. Viktor Lazić from the "Adligat" Society, Mr. Jovan Hans Ivanović and his "Mihailo Petrović Alas" Foundation, "Mihailo Petrović Alas" Primary School, "Svetozar Marković" University Library, Belgrade City Museum, Zavod za udžbenike (Institute for Textbook Publishing) in Belgrade, Virtual Library of Faculty of Mathematics in Belgrade and Digital Legacy of Mihailo Petrović Alas.

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Finally, we would like to express our gratitude to Mr. Mirko Milićević from the publishing house "Dosije Studio" for excellent prepress preparation of the monograph.

S. Pilipović, G. Milovanović, Ž. Mijajlović

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EDITOR'S FOREWORD

As soon as one first encounters the work of Mihailo Petrović, it becomes evident that he was a person that according to its numerous traits was a polymath. Above all, the academician Petrović was a gifted mathematician and a renowned professor at the University of Belgrade, but also a fisherman, writer, philosopher, musician, world traveler and a travel writer. He earned a degree in mathematics at the Belgrade Grand School and a licentiate degree in mathematics, physics and chemistry at the Sorbonne. At the age of 26, only a year after he had completed his studies, he defended his PhD degree in mathematics at the same university, as a student of the famous French mathematicians Henri Poincaré, Charles Hermite and Charles Émile Picard. In the same year (1894) he was elected to the position of professor at the Grand School to which he brought the spirit of the French mathematical school. It was at that point that his long and prolific journey through science began, whereas, owing to him, Belgrade achieved parity with other major European centers in mathematical sciences. He became an initiator and a leader of the Serbian mathematics and strongly contributed to the spirit of the modern European science in Serbia.

Petrović's expertize spanned several mathematical areas in which he achieved scientific results of world-class relevance: differential equations, numerical analysis, theory of functions of a complex variable and geometry of polynomials. He was also interested in natural sciences, chemistry, physics and biology, and he published scientific papers in these fields, too. In his scientific endeavor he managed to meet the most rigorous standards of the most developed European countries. In a brilliant rise, in a few years' time, up to the early 20th century, he wrote around thirty papers that he published in the leading European mathematical journals. It was due to this fact that he was elected a member of the Serbian Royal Academy as early as at the age of 30, and soon after he became a member of a number of foreign academies and prominent expert societies. He won the greatest respect of the global mathematical community: he was among few mathematicians (13) who delivered at least five plenary lectures or lectures as a visiting lecturer at the International Congress of Mathematicians (ICM). He delivered five such lectures (1908, 1912, 1924, 1928 and 1932). One such invitation has been considered by the mathematical community as an equivalent of an induction to a hall of fame. In addition, it has been considered that Petrović was a founder of new scientific disciplines, namely mathematical phenomenology and spectral theory. He invented several analogue computing machines, possessed technical patents and was the main cryptographer of the Serbian and Yugoslav Army.

Up to the Second World War he was the mentor of all doctoral thesis in mathematics defended at the University of Belgrade. Aforementioned is related to one of professor Petrović's greatest and most important achievements – he was a founder of the Serbian mathematical school that has produced a great number of renowned and successful mathematicians not only in Serbia but also around the world.

In 2018, the Serbian Academy of Sciences and Arts and mathematicians in Serbia celebrate the 150th anniversary of the birth of Mihailo Petrović Alas. Throughout this year, the Academy has organized a large exhibition dedicated to Petrović, alongside a solemn gathering and a conference. This monograph commemorates this important jubilee of the Serbian mathematics. Given the fact that a lot of articles on Petrović have already been written, and that his collected works were published at the end of the last century, the editors and authors of the papers in this monograph were faced with a daunting task of finding some new details from professor Petrović's life and career. Even more so given that his body of work is immense, spanning different scientific areas and encompassing topics that at first glance one finds difficult to combine. As Dragan Trifunović, Petrović's biographer and a man who most thoroughly studied his life and work, noted on one occasion that almost an institute was necessary that would encompass professor's entire body of work. Therefore, we set a relatively modest goal to ourselves to shed light upon some main points of Petrović's life and work, times and circumstances he lived in, as well as to elaborate on the present developments in relation to the Serbian mathematical school, through a selection of papers. The authors of the papers steered clear of technical details and excessive use of mathematical language. Hence, the monograph is intended for a broader readership, in particular to those readers who are interested in the history of Serbian science and its evolvement at the turn of the 20th century, but also to those who want to gain a deeper insight into the life of a brilliant mathematician and a polymath, and, we can quite freely say, an unusual personality.

Ž. Mijajlović, S. Pilipović, G. Milovanović

MIHAILO PETROVIĆ ALAS: LIFE AND WORK

ACADEMICIAN MIHAILO PETROVIĆ – HIS CONTRIBUTIONS TO SCIENCE AND EDUCATION

– 150 years later –

Stevan PILIPOVIĆ
Serbian Academy of Sciences and Arts
University of Novi Sad, Faculty of Sciences

Academician Mihailo Petrović passed away 75 years ago, aged 75. This year we mark the 150th anniversary of his birth. We shall attempt to answer several questions in this paper, most notably why professor Petrović, who became an academician aged 31, is so important for our mathematics and science in general? Why is Mihailo Petrović so loved and esteemed among our people?

A lot has been written about academician Mihailo Petrović, mainly about his mathematical results, travelogues and philosophical treaties and essays. There is ample literature about his friendships with fishermen, numerous fishing adventures, "Suz" music orchestra and his bohemian life. Almost all his students wrote about him, particularly professors Dragoslav Mitrinović, Tadija Pejović and Radivoj Kašanin. Within commemorative programmes organised to mark his birth anniversary, our numerous mathematicians analysed almost all his works. One of our greatest scientists, academician Milutin Milanković and our famous seismologist Jelenko Mihailović, who was Petrović's peer and friend, depicted Belgrade in their authentic notes[31], in the years between the two world wars and Mihailo Petrović as the icon of Belgrade's scientific and social life in the late 19th and the first half of the 20th century.

The work of our eminent historian of mathematics Dragan Trifunović also contains plenty of information. In 1998, the Institute for

Textbook Publishing and Teaching Aids, in cooperation with the Faculty of Natural Sciences in Belgrade and the Society of Mathematicians of Serbia, published an outstanding book – *The Collected Works of Mihailo Petrović*, in 15 volumes[32]. Trifunović headed the editorial board of this publishing project, which sheds light on the personality and work of Mihailo Petrović. Our most eminent academicians participated in the preparation of *The Collected Works*. Another important work, a forerunner of *The Collected Works*, is *The Chronicle of the Life and Work of Mihailo Petrović, 24 April 1868 – 8 June 1943*, by the same author. Trifunović collected available archival data, classified by year Petrović's biography and arranged his letters, scientific works, descriptions of his travels and other events from his life. Particularly interesting are the letters reflecting Petrović's personality and his line of reasoning. Original notes fill the reader with excitement and anticipation as one can feel the spirit of the time in which Mihailo Petrović lived and worked, and gain a genuine picture about him as a modest man of great capabilities. We have today all of his works, owing primarily to professor Žarko Mijajlović and his associates who digitalised all the books concerning Mihailo Petrović.

Mihailo Petrović Alas, or our Mika, as his contemporaries called him, was a fisherman and seafarer, musician and thinker, writer and bohemian, but, first and foremost, a great mathematician. He was highly enduring and physically strong, which enabled him to participate in numerous adventures and travels. For instance, he survived malaria at his last transoceanic travel, during an expedition in the southern part of the Indian Ocean. He was already 65 at time. According to him, he survived by drinking only water and mild wines. From this distance, I have the impression I am writing about an exceptionally interesting man of a great creative talent, a genius, who wove events from ordinary and everyday life into uninterrupted creative thought, which beset him wherever he was and whatever he did. I believe his activities outside mathematics were the periods of relaxation, during which he still ruminated about mathematics and philosophy, or practical solutions of problems, such as patents. Exceptional creativity and universality are the main features of Mihailo Petrović's scientific work. By carefully examining Petrović's activities, including fishing, one can discern his outstanding creative spirit. Our enthusiasm with academician Petrović is based on our understanding of his simplicity and modesty, as well as, given the level of mathematics at the time he lived in, his achievements in mathematics. Petrović was one of the most educated mathematicians Serbia has ever had.

This paper focuses on Petrović's scientific and teaching activities. In this light, we also examine today's achievements in the field of analysis within the so-called Novi Sad School of Analysis. Other mathematical fields that Petrović dealt with will be presented by our eminent colleagues. The literary, historical, travelogue, ethnographic and professional works on fishing and music will be described in other sections of this monograph.

Mihailo Petrović Alas (“Mihailo Petrović Alas” Primary School, Belgrade)

MATHEMATICS IN SERBIA IN THE SECOND HALF OF THE 19th CENTURY

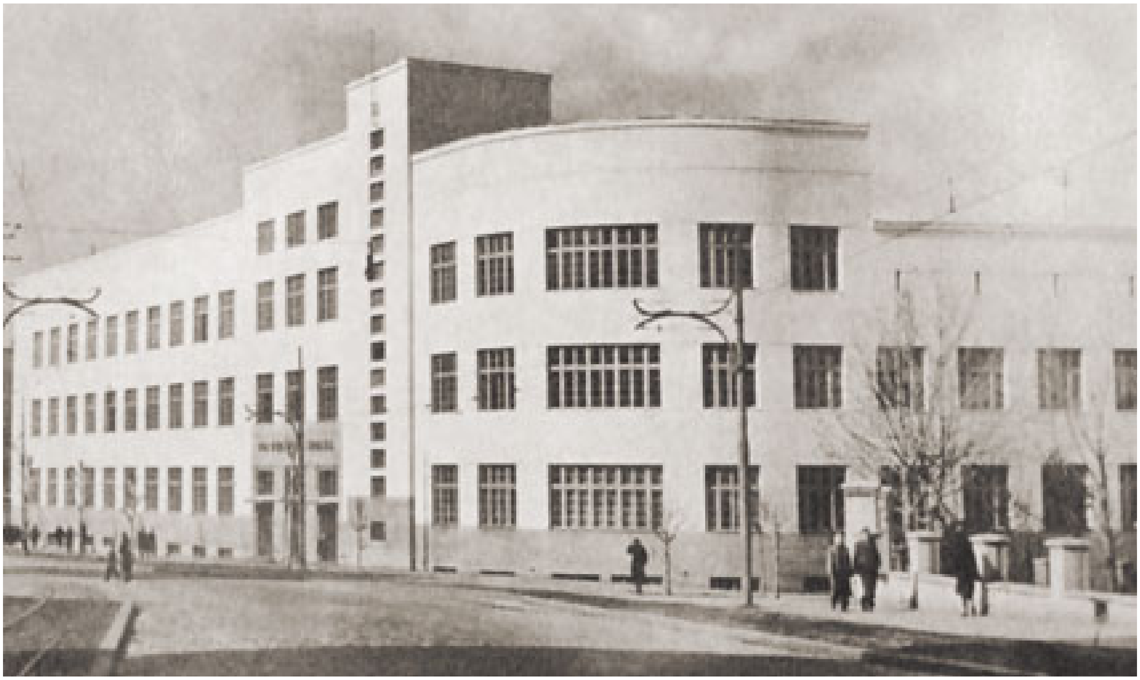
In the 19th century, from the establishment of the *Society of Serbian Letters*, the Serbian language was the dominant theme of the most educated people in Serbia. Natural sciences and mathematics were in the background given the lack of educated mathematicians. However, in addition to members of the *Serbian Learned Society* and professors of the Great School Dimitrije Nešić and Bogdan Gavrilović, there are several other professors of the Great School and authors of the first university mathematics textbooks in our country that are worth mentioning: Atanasije Nikolić, the founder of the Society of Serbian Letters (together with Jovan Sterija Popović), Emilijan Josimović, also a member of the Society of Serbian Letters, Dimitrije Stojanović and Petar Živković, members of the Serbian Learned Society. Professor Nešić, later Member of the Serbian Royal Academy, began his career with an article on squaring the circle in 1878. Nešić's ideas were of much higher quality than the results were not possible to be proved, as earlier established by Hermite and Lindeman. Nešić also wrote several interesting works, printed in Serbian in *Glasnik* of the Serbian Royal Academy.

Some of the world's most brilliant mathematicians were active at the time – for instance, Bernhard Riemann, Sophus Lie and Henri Poincaré. Is it necessary or even possible to compare the situation in Serbia with that in France, Russia, Germany, Italy, Holland, Austria, Switzerland or Hungary? Nonetheless, in the last decade of the 19th century, some great scientific minds started their work in Serbia, such as Jovan Cvijić, Ivan Đaja, Jovan Žujović, Sima Lozanić, Kosta Stojanović, Đorđe Stanojević, Slobodan Jovanović, Branislav Petronijević and mathematician Mihailo Petrović. This is why such a comparison in the late 19th century is, after all, possible.

Around ten years before these great names of our science appeared in Serbia, we had two extraordinary figures in America – one of the greatest innovators in world history, Nikola Tesla, 12 years older than Mihailo Petrović, and Mihajlo Pupin[33], a great scientist and professor, 14 years older than Petrović. The two of them already had exceptional careers, demonstrating supreme talent that we take pride in and that inspires us with the awareness that we can create great deeds.

Let us return to the description of the development of mathematics in Serbia at the time. Above all, let us highlight the merits of our professors: Dimitrije Nešić, Petrović's mathematics professor at the Great School before he left for France, and Petrović's colleague and friend Bogdan Gavrilović, a member of the Serbian Royal Academy and its later president. Academician Miodrag Tomić gives us plenty of selected information about Dimitrije Nešić and his merits for mathematics in his article *Contributions about Mathematical Sciences among the Serbs* (Mathematical Institute, 1992). Professor Gavrilović (also Rector of the University of Belgrade) was extremely important for the development of higher education in Serbia, though in scientific terms he remained in Petrović's shadow. A distinguished pedagogue, writer of several solid and studious mathematical higher education textbooks, Bogdan Gavrilović occupies an important place in our history, i.e. the most important after Mihailo Petrović, in the late 19th and early 20th century.

Historical figures in all fields of human activity are formed in certain time periods and favourable social and economic circumstances. The geniuses of science, art and invention live in any time and in all parts of the world. However, historically the most important personalities in any creative field, in addition to being ingenious, must be lucky to be born at the right moment. Namely, I wish to highlight an almost concurrent appearance of several historical figures in our scientific and cultural milieu by the late 19th century. We are proud of all of them, although with the enormous development of science their achievements are today merely smaller elements of highly developed and advanced theories. It is in this context that we observe academician Mihailo Petrović Alas, one of the most important scientists in Serbia and the founder of the mathematical educational and scientific system in Serbia.



First Belgrade Gymnasium, XIX century

FRAMEWORK OF SCIENTIFIC WORK OF MIHAILO PETROVIĆ

The scientific and pedagogical activity of academician Mihailo Petrović marked the path in the development of mathematics and university teaching of mathematics in Serbia. His biography is an important contribution in understanding historical events in Serbia in the late 19th and the first half of the 20th century.

After completing the Department of Natural Sciences of the High School in Belgrade, in 1985–1989 Mihailo Petrović, owing to favourable circumstances and his enormous talent that must have been evident, became a student in Paris at the most opportune time. He gained diplomas at Sorbonne in Paris, first in mathematics, and later in physics. A diploma on completed studies of chemistry was also recently found in his legacy. He was one of the first foreign doctoral students in mathematics at l'École normale supérieure. This school was and still is the most famous educational mathematical centre, whose lecturers were some of the greatest mathematical minds from the late 19th and the first half of the 20th century: Jules Henri Poincaré (1854–1912), Jean-Gaston Darboux (1842–1917), Paul Appell (1855–1930), Paul Tannery

(1843–1904), Charles Hermite (1822–1901), Paul Painlevé (1863–1933). Painlevé served twice as the Prime Minister of the Republic of France – during and after World War I. Petrović's friends were his peer Félix Édouard Justin Émile Borel (1871–1956) and around ten years younger Paul Antoine Aristide Montel (1876–1975), both of them exquisite personalities of world science. In the company of the greatest mathematicians whose originality of scientific work was their main feature, Mihailo Petrović demonstrated all his qualities. By defending his doctoral dissertation in 1894 before the Dissertation Committee consisting of Hermite, Picard and Painlevé, and having published a paper in the French Academy's journal, the famous *Compte Rendus*, Mihailo Petrović entered the world of great mathematicians of his time and wrote a number of important papers. He dedicated his doctoral dissertation to Tannery and Painlevé.

According to the information analysed by Trifunović, academician Mihailo Petrović published 393 works, of which 328 are mathematical manuscripts from twelve different areas, according to Trifunović's categorisation, in the leading world journals both then and today. He published 30 papers in *Compte Rendus*, where reports of French academicians on mathematical manuscripts considered important were printed. Even more important are the following journals where Petrović was publishing: *Acta Mathematica*, *Mathematische Annalen* – two papers, *Bulletin de la Société Mathématique de France* – fourteen papers, and *American Journal of Mathematics* – three papers. He also published in numerous other journals, primarily in Switzerland, Germany, the Czech Republic and Poland. Information about the number of papers is not reliable given that Petrović published some papers both in Serbian and French. In any case, Petrović was one of our most prolific mathematicians, whose papers were published in the journals still highly ranked on the lists of mathematical journals.

To better understand the time in which Mihailo Petrović worked on his doctoral dissertation, we should underline that in the 1880s and 1890s the greatest minds of French and world mathematics, Picard, Painlevé and Fucs studied nonlinear second-order equations with immovable branch points. The concept of the Painlevé transcendent, which is still highly topical, derives from the analysis of special functions arising as solutions of specific classes of nonlinear second-order differential equations in the complex plane. Among them are elliptic functions (doubly periodic functions), which are one of the most important classes of special functions. Painlevé transcendents are defined by nonlinear second-order differential equations with solutions whose singularities have the Painlevé characteristic: the only movable singularities are poles. Painlevé, followed by Fucs and Bertrand Gambier (1910) described the equations of this type – the so-called Painlevé equations. For the equation

$$y'' = F(x,y,y')$$

where *F* is the quotient of two polynomials under *y* and *y'* with coefficients which represent holomorphic functions, Painlevé found fifty general forms with immovable branch points and reduced them to six essentially new equations known as the Painlevé equations. After Fucs and Gambier supplemented his results, the solutions of these equations are known as the Painlevé transcendents. These six generic cases have a great importance today and are applied in statistical mechanics, physics of plasmas, theory of nonlinear waves, quantum gravity theory, quantum field theory, relativity theory and nonlinear optics. They are non-reducible in the class of classical special functions and have a number of exceptional characteristics. Integrable systems are reduced to these types of equations. For instance, the solutions of the soliton type of nonlinear differential equations are reduced to the Painlevé equations by the method of inverse scattering. Let us mention that this problem is being examined, by applying the methods of algebraic geometry, a modern mathematical field, by our colleagues in the group of professor Vladimir Dragović. Several their papers dedicated to the Painlevé equations have been recently printed or are about to be printed in the leading world journals. The theory related to the Painlevé equations is today unusually rich and modern. In 2017, the American Institute of Mathematics in San Jose organised a high-level conference devoted to the application of the Painlevé equations in the study of random matrices and the number theory. I highlight all this because I believe that the works of Mihailo Petrović, and probably those of his students, contain the forgotten original roots of modern mathematical research.

Painlevé published his first results in 1887 and 1895. Besides him, Poincaré and Picard worked on this issue. The problem of three bodies, linked to the movement of planets, was (almost) solved by Poincaré. This is worth emphasising because, I reiterate, Mihailo Petrović defended his doctoral dissertation before the Dissertation Committee consisting of Picard and Painlevé. Let us mention that Poincaré participated in developing the theme of the doctoral dissertation. Even Poincaré's biography contains information that Mihailo Petrović was his student. The third member of the Committee, Hermite, as the oldest and highly respected professor, a mentor to Poincaré, Tannery, Stieltjes, considerably contributed to the reputation of this Committee.


THÈSES

A LA FACULTÉ DES SCIENCES DE PARIS

PARIS.


PREMIÈRE PARTIE.

CHAPITRE I.

Book covers and the first page from Petrović's doctoral dissertation ("Svetozar Marković" University Library)

DOCTORAL DISSERTATION

Mihailo Petrović's doctoral dissertation deals with the immovability of zeros, poles and essential singularities of solutions of particular classes of algebraic first-order and second-order differential equations. Let us explain in short the movability of zeros and singularities of the general solution of a differential equation which, in a general case, depends on the constants directly calculated from initial conditions. First of all, let us remind that singularities are poles, essential singularities, finite order and logarithmic branch points. Movability means that zeros and singularities change incessantly with a change in initial conditions, i.e. it is a "well-posed" problem in contrast to "ill-posed" problem which often cannot be solved if initial conditions change a little. The adjectives "ill" and "well" do not have a formal meaning in mathematics, and in this particular case they only testify to the constant dependence of solutions on initial conditions. The problems studied by Mihailo Petrović are those in respect of which the solutions of equations have singularities independent of a change in initial conditions.

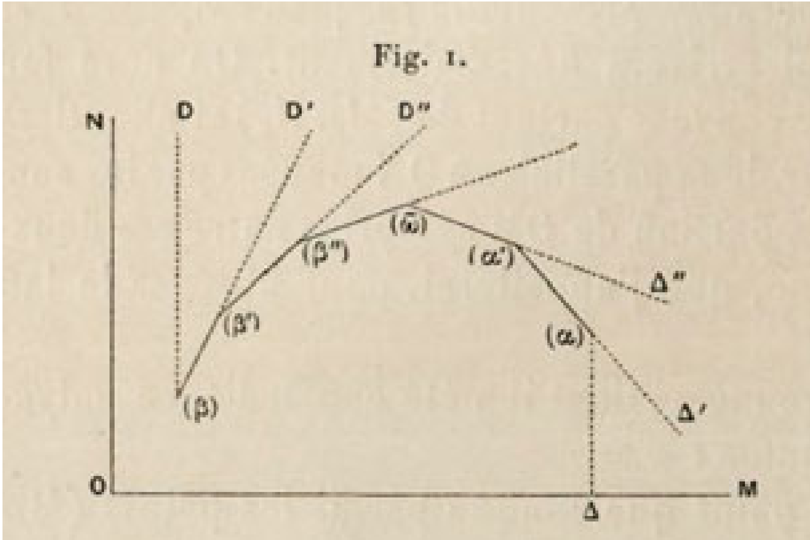
In the first part of his doctoral dissertation, Petrović analyses first-order nonlinear equations with products of powers of derivative of an unknown function y', function y, and holomorphic function $\varphi_i(x)$ under the independent variable x. Their shape is the following:

$$F(x, y, y') = \sum_{i=1}^{s} \varphi_i(x) y^{m_i} y'^{n_i} = 0.$$

In this analysis, Petrović constructs the polygon added to the following equation with vertices

$$(M_i, N_i),\ N_i = n_i,\ M_i = m_i + n_i, \qquad i = 1, \ldots, s.$$

He arranges them according to specific rules, with (M_α, N_α) and (M_β, N_β) signifying the most distant/proximate vertices to the axis ON, while λ designates the coefficients of the direction of



Polygon R, a drawing from Petrović's dissertation ("Svetozar Marković" University Library)

the sides of the polygon. In the first chapter of the first part of the thesis, Petrović gives a complete answer to the defined problem, by analysing the given convex polygon. For the infinities (as Petrović calls poles and essential singularities) of the general solution (integral) of the first-order equation $F(x,y,y) = 0$ not to change with the integration constant, it is necessary and sufficient that the polygon joined to F does not have any vertex to the right from the most highly elevated vertex of the polygon. The existence of the movable zero of order λ is necessary and sufficient for the polygon to have a side with the slope λ while for it to have a movable pole of order λ it is necessary and sufficient for the polygon to have a side with a slope $-\lambda$. Based on these two out of six assertions from the first chapter, we can clearly see the simplicity of Petrović's formulations.

Such geometric interpretations are more easily acceptable (let us remember Newton's polygons in the case of partial differential equations) and essentially give a new geometric method for the analysis of the solution of the equation. The conditions for essential singularities are also given. There may be a finite number of poles and essential singularities in the case of their immovability. In the second chapter, Petrović explains the application of theorems from the first chapter, connecting them to the results of Painlevé and Fucs which arise from immovability of branch points. Let us mention an assertion concerning the rational function R and equation $y' = R(x,y)$: this equation may not have more than three different uniform (single-valued) solutions, which I will hereinafter call only solutions. If there are three of them, it is Riccati's. If there are two of them, it is linear or Riccati's or has a solution of a particular shape. If it has one solution, it is reduced to one of the earlier forms or a special form. We do not mention special forms.

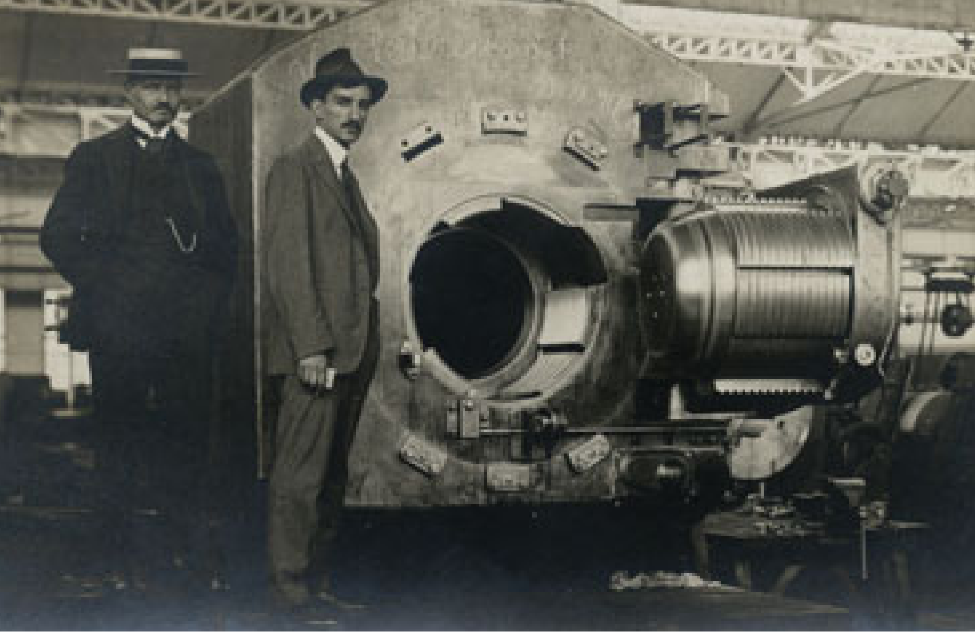
For precise formulation of these assertions, see the paper by professor V. Dragović in this monograph. Furthermore, in the second part of the first chapter, he examines various forms of the right side of the equation, function $R(x,y)$, and gives general forms of the solutions of equations.

In the second part of the thesis, Mihailo Petrović deals with nonlinear ordinary differential equations of the second order, with similar questions as in the first part. He deals with the movability of the singularity of the equation and gives sufficient conditions for the immovability of poles, essential singularities and zeros, under the assumption that branch points are immovable. Starting from the results of Poincaré and Fucs and by drawing polygons under the same principle as in the first chapter, though in a much more complex form, Petrović shows his results rather than illustrations, as sufficient conditions, and less as necessary and sufficient conditions in the form of a theorem. The necessary conditions in the mathematical sense were achieved only in special cases. In the second chapter of the second part of his doctoral dissertation, Mihailo Petrović gives examples of solutions through holomorphic solutions of adjoint equations. Under appropriate conditions, starting again from the results of Painlevé and Fucs about the immovability of branch points, he finds functional links, the so-called first integrals that the solutions must meet and specifies the number of solutions.

Academician Bogoljub Stanković translated Petrović's dissertation into Serbian. This translation is part of the first volume of *The Collected Works*, prepared by academician Stanković.

DIFFERENTIAL EQUATIONS

The turn of the 19th into the 20th century brought great discoveries in almost all sciences, particularly mathematics. New mathematical theories and new methods emerged. Though at the time it might have looked different, but the events that happened have their repercussions still today. Mihailo Petrović belonged to the generation of the greatest minds in the field of the analytic theory of differential equations, which at the time reached its pinnacle, while on the other hand, new mathematical disciplines emerged, which were theoretically highly demanding, including the theory of partial differential equations. Mihailo Petrović was at the height of this mathematical field, the most developed at the time, and it is in this light that we should observe the exceptionally high scientific level of his research at the time when he was writing his doctoral dissertation, including the destiny of these results as well. This is worth emphasising because generations of our mathematicians who dealt with differential equations did not go much further than the results of Mihailo Petrović from the period when he defended his doctoral thesis, or from the works he wrote in the first twenty years of the 20th century. The majority of his doctoral students explored in their theses the topics relating to ordinary nonlinear differential equations, mainly of Riccati type, or with a qualitative analysis of some classes of equations. It is possible that the basic problem was the fact that Mihailo Petrović's successors, and later his students in the fields of differential equations, perhaps less followed the development of mathematical physics or other branches of mathematics, where the results of Painlevé and Picard remained dominant. Besides, insufficiently followed is the theory of partial differential equations, which at the time was developing exceptionally fast both in theoretical terms and in terms of application in almost all natural and technical sciences. Petrović himself did not follow new directions of development of the theory of partial differential equations with entirely new methods arising from the then modern areas – set and algebraic topology, geometry and algebra, particularly functional analysis. The results of Hilbert, Lebesgue, Dirac, Banach, Sobolev and many others brought entirely new insights into mathematical research and its link with other scientific areas. The progress and development of mathematics was particularly intensive after World War I. One should bear in mind that Mihailo Petrović was aged around 50 at the time, and was increasingly outside scientific developments due to complicated circumstances in Serbia in the post-war period. Some of our colleagues mathematicians unjustifiably wrote that Petrović, with his exceptional energy in the areas that he dealt with, hindered the development of other, also important mathematical fields. It is hard to provide arguments, particularly from this perspective, outside the time during which Mihailo Petrović lived and worked. He did not encourage the areas he was not familiar with, nor did he impose upon anyone doctoral dissertations relying on the research that he personally engaged in.



Mihailo Petrović with Prince Đorđe Karađorđević (Foundation "Mihailo Petrović Alas"/ Primary School "Mihailo Petrović Alas", a gift from Jovan Hans Ivanović)

SCIENTIFIC WORKS – DIFFERENTIAL EQUATIONS

The first work of Mihailo Petrović signalled the problems explored in his doctoral dissertation, while the works that followed the dissertation relied on its results. The independence of singularities, zeros, extremes or other features of a general solution with respect to constants, dominates in all these works as one of the determinants of Petrović's scientific work. This is the essential, constitutive feature of a model related to an equation describing the model. When analyzing the residue of the function and exploring the so-called binomial first-order equations, i.e. the asymptotic solution, or when writing about a "class of second-order differential equations" or the nature of the solution, he aims to examine the internal structural relationship between the dependent and independent variable given by equations. This can also be seen in some of his works reprinted in the first volume of *The Collected Works*. A great creative potential in his scientific papers initiated by his doctoral dissertation is also seen in the works written and published during World War I, when his papers in *Compte Rendus* were presented by Picard, Appel and Hadamard. During the war, he was dispatched to Switzerland, in service of Prince Đorđe Karađorđević, to be a cryptographer at the Serbian Military Command. Even there did he have sufficient strength to tackle serious mathematical problems. It is interesting to note that his paper in which he described his own most important results in the period until 1922 was published in France. This analysis and overview of Petrović's results, written by academician Bogoljub

Stanković, clearly attest to the quality and value of results presented in the doctoral dissertation. An excellent overview of Petrović's scientific and teaching opus in the article by academician Miodrag Tomić, in the first volume of *The Collected Works*, clearly confirms all already mentioned qualities of Mihailo Petrović as a scientist and a renaissance personality in our science.

According to Dragan Trifunović's statistics, in addition to his doctoral dissertation, Mihailo Petrović published 86 papers on differential equations. In the first volume of *The Collected Works*, 14 papers were reprinted and translated into Serbian, primarily in the field of the analytic theory of differential equations. The second volume contains 23 papers and the monograph *The First Definite Integrals*. Those were mainly works in the field of the qualitative analysis of general linear and nonlinear equations, and typical solution of some classes of equations. Papers are presented chronologically so that a careful reader can follow the development of ideas and the quality of publications compared to already printed manuscripts.

In the works published in the second volume of *The Collected Works*, Petrović studied the qualitative features of solutions of differential equations. He upgraded his new ideas based on those from his doctoral dissertation. In this context, he particularly examined various forms of the Riccati equation $z'(t) = a(t)z(t) + b(t) + c(t)z(t)^2$ and generalisations which can be solved with integrations and known special functions. He based the classification of differential equations on appropriate transformations of independent and dependent variables. Petrović used his exceptional knowledge of the theory of analytic functions. He thus found canonical forms of various classes of equations and solved them. The analysis of equations through the formulation of sufficient conditions for equation coefficients, along with appropriate initial conditions, is the domain of the qualitative analysis of differential equations to which the majority of Petrović's papers are devoted in the second volume of *The Collected Works*. The theorems about the comparison of solutions of equations relative to the comparison of coefficients or right-hand sides of equations, the so-called Sturm-type theorems, were the inspiration to his students in their doctoral dissertations. He studied Champling-type equations independently of Champling, and before him, although he did not go into the detailed analysis which was later formulated. Applying subtle mathematical insight and having clear motivation, he obtained results of general character for various classes of linear and nonlinear equations. The solution asymptotics also has an important role in the analysis of equations – those were again mainly nonlinear, Riccati-type equations. When reading the papers from the second volume of *The Collected Works* which Mihailo Petrović published aged 50 and 60, one can see a decline in scientific enthusiasm, although these papers still contained new ideas of qualitative generalisations and classification of equations.

Unlike today's practice, Mihailo Petrović wrote his works by explaining a theory starting from special to general mathematical conclusions. As if in a story, easy conclusions are given and generalised; then, fully explained, they are turned into assertions that are only at the end formulated. This enables the reader to easily follow the narrative, with clear objectives that are already defined at the beginning. We can add another analysis of Mihailo Petrović's works, written by himself, to our assessment of the quality of papers from the second volume of *The Collected Works*. It is worth noting that professor Ljubomir Protić wrote a systematic article in the same volume related to Petrović's work. Together with professor Milorad Bertolino, he was one of the important followers of Mihailo Petrović in the theory of ordinary nonlinear differential equations.

APPLICATION OF EQUATIONS – PHENOMENOLOGY

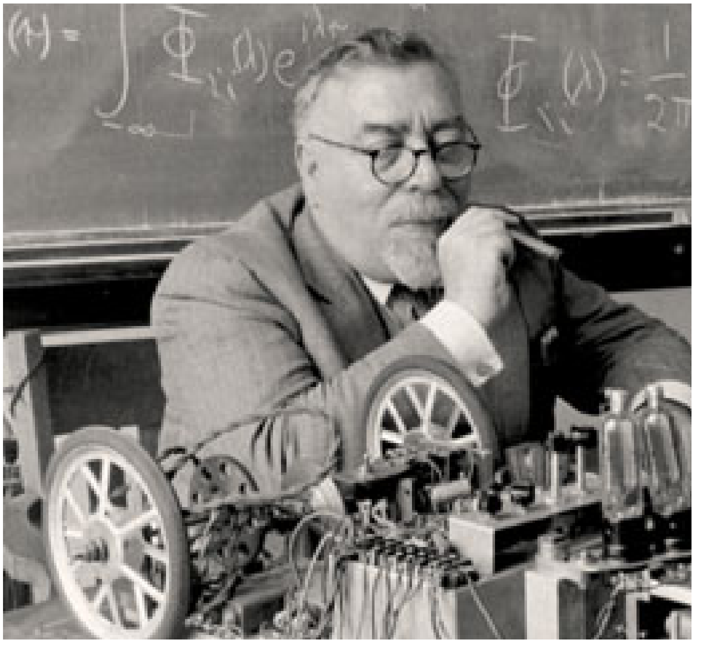
The seventh volume of *The Collected Works* contains the equations of mathematical physics in the monograph *Elements of Mathematical Phenomenology* (later in the monograph *Phenomenological Reflections*). The results of modelling explained by Petrović in philosophical sense are a highly important scientific contribution in applied mathematics. Moreover, it was in philosophy and other areas of humanities that his phenomenology with metaphors and allegories obtained a number of followers.

As an exceptionally educated mathematician, Mihailo Petrović dealt with mathematical research which can be divided, perhaps too freely, into *l'art pour l'art* research and motivated, i.e. applied research.

In a rather free interpretation, *l'art pour l'art* research concerns his doctoral dissertation and differential equations which he theoretically examines from the viewpoint of the theory of analytic functions, power series and appropriate algebraic problems, which he links to the complex structure of analytic functions.

Applied research is related to modelling, i.e. formulation of models, various physical or chemical phenomena. Moreover, in phenomenology to which he devoted a great number of papers and which he experienced as the most important part of his research, Mihailo Petrović tried to determine, through analytic dynamics, "the activity of causes" by establishing appropriate systems of equations and thus describe phenomena independently of the nature of objects and phenomena being modelled. Phenomenology arises from his algorithmic approach to mathematical problems. A monograph on this topic was printed by the Academy in Serbian in 1911. Petrović published the same work in an abbreviated version in Paris, at his own cost. Petrović analysed general phenomena arising from the minimum number of basic facts, causes, established analogies and arrived at conclusions based on a narrow, base set of data. This is a generally accepted procedure in mathematical research.

Metaphors and Allegories (a posthumously published monograph) elaborates on the ideas of phenomenological research. As unlike phenomenology, metaphors and allegories are significantly void of mathematical formulae, they offer, in addition to philosophy, a significant basis for the structural research of phenomena in other social and humanistic disciplines as well. For instance, in the phenomenology of Mihailo Petrović, in his metaphors and allegories, we also



Norbert Wiener (1894–1964)

find the contemporary line of linguistic research. Almost a hundred years after the publication of Petrović's works, Noam Chomsky, a foreign member of our Academy and the greatest world linguist, wrote that analogies or the establishment of similarities represent the key issues of language functioning. Academician Jasmina Grković-Major pointed out this and referred me to an interesting article by Dr Ivana Bašić from 2012.

Our literature perhaps overemphasises the fact that Mihailo Petrović was one of the first creators of phenomenology. There were before him many other authors of scientific theories attempting to use mathematical clarity and an axiomatic approach to the examination of general laws. Many our mathematicians believed that Petrović's phenomenology is the precursor of cybernetics. Dragan Trifunović's dissertation "Study of Modelling in the Work of Mihailo Petrović" dealt with phenomenology, metaphors and allegories. Somewhat younger than Petrović, world-renowned mathematician Norbert Wiener is considered the father of cybernetics. It is a real pity that the works of Mihailo Petrović were insufficently known at the famous Massachusetts Institute of Technology, where an entire group of brilliant mathematicians worked, headed by Wiener.

Let us emphasise that Petrović's descriptions of phenomena through the systems of relevant equations today still constitute a modern approach to exploring phenomena in nature, particularly in the context of the so-called motivated or applied mathematics, as the most important determinant of contemporary mathematical studies.

OTHER MATHEMATICAL WORKS

As we have already emphasised, the most important works of Mihailo Petrović belong primarily to the fields of analysis, differential equations, complex and real analysis.

Within the theory of analytic functions, he explored functions whose Taylor series do not have zeros in an appropriate circle of convergence. Landau, Hardy, Fejér, Montel, Pólya studied his works in this field, while Jentzsch developed them in his doctoral dissertation. Related to these works are his papers in the field of algebraic equations. Particularly interesting is his work on the geometry of polynomial zeros, which was also studied by this group of renowned mathematicians. Petrović determined the ring in which an algebraic equation has at least one root without using the Rouché's theorem. His paper from 1899, published in *Compte Rendus*, was the first paper determining the number of zeros contained in a given circle.

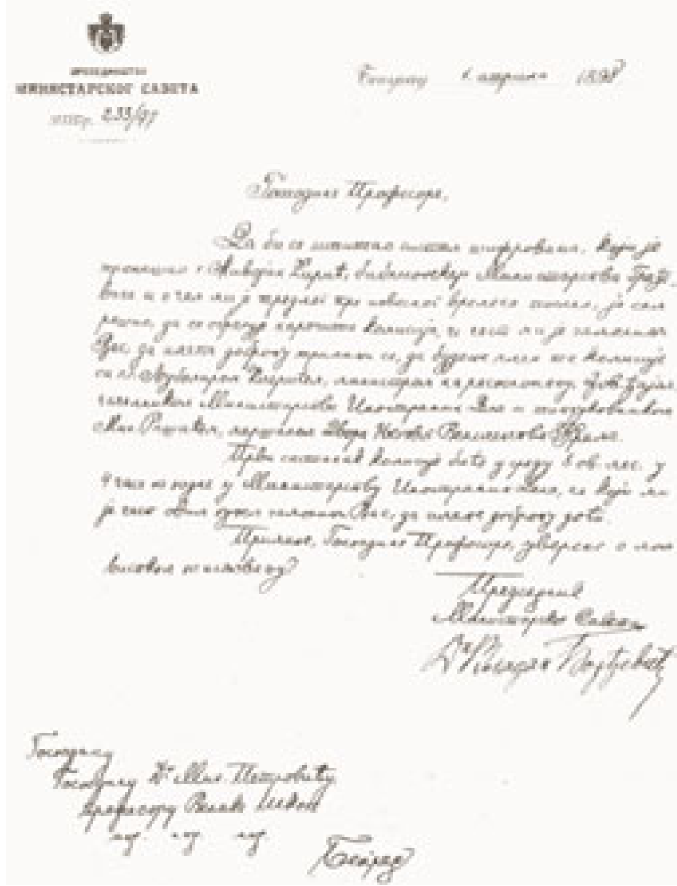
Let us also mention his contribution to numerical mathematics – for instance, the analysis of number differences and the instability of solutions, i.e. the interval analysis separately published in his textbook *Calculation with Number Differences*. Related to this research are his papers in which he dealt with the calculation of specific integrals through series, to which he devoted a particular textbook. In addition, his textbook *Elliptic Functions* is still popular today.

In an analogy with light spectres in physics, Petrović developed the theory of mathematical spectres which is not related at all with today's spectral theory of operators. Quite the contrary, it can be classified into the theory of numbers and cryptology – algorithmics, rather than analysis. During one term at Sorbonne, he held lectures on spectres, and published two monographs in this field in Paris. The main idea behind this theory is that infinite series of data are coded with infinite decimal numbers (with zeros and ones), translating mathematical operations with data into certain numerical or combinatory procedures. Relying on his exceptional knowledge of the theory of function series, he assigns to the appropriate analytic function the analytic expression – a number which with its decimals determines this function, i.e. its Taylor coefficients. His student, professor Konstantin Orlov focused later on this field, in his dissertation. Orlov was one of the most renowned followers of professor Petrović in the field of differential equations and numerical analysis. Unfortunately, the spectral theory did not find a significant place in mathematical literature. It seems to have appeared too early.

Petrović's inequality

$$\sum_{k=1}^{n} f(x_k) \leq f(\sum_{k=1}^{n} x_k)(n-1)f(0)$$

is the precursor of Jensen's inequality. It is related to the study of convex functions and was an inspiration to professor Mitrinović and his students in the examination of analytic inequalities and writing of a number of exceptionally well-received and cited books on functional equations and inequalities.


МИНИСТАРСКОГ САВЕТА

An invitation to Mihailo Petrović for participating in the work of the Encryption Commission, 1 April 1898

While serving in the army during the Balkan Wars and the Great War, Petrović dealt with cryptography and gained the rank of a reserve engineering lieutenant colonel. From the Balkan Wars until the start of World War II, he was a cryptographer at the Serbian army. Petrović took a lot of pride in his military rank.

As already said, Petrović completed in Paris the studies of physics and chemistry as well. He liked chemistry a lot. He probably attended lectures of Sima Lozanić, a professor at the Belgrade Great School and later an academician. He expressed particular creativity in his works in which he formulated models for various phenomena in mechanics and chemistry, Based on simple models, he constructed devices that produced solutions to equations through mechanical devices or chemical reactions. Analogous computers, and present-day cutting-edge quantum computers, were created based on the principles of electrical and physical-chemical reactions, which is the area of the state-of-the-art theory in computer science today. A possible link, in a rather free interpretation, with the ideas of modern quantum computers can be found in Petrović's results.

At the start of his career, along with mathematical modelling, Mihailo Petrović dealt with the creation of patents, range markers and depth gauges. He constructed the gear and eternal calendar. In 1900, he received the third award at the World Exhibition in Paris for the hydro integrator described in the *Journal of the American Mathematical Society* in 1898.

THE ECHO OF PETROVIĆ'S SCIENTIFIC RESULTS

There are few persons today who are well familiar with Petrović's doctoral dissertation and works. Objectively, mathematical results last as long as they are topical in contemporary research. After some time, only a small number of them become classical results and enter textbooks where authors are no longer mentioned, unless they carry the authors' names, as is the case, for instance, with the Pythagorean theorem, Newton-Leibniz formula or Petrović's inequality.

The works of Mihailo Petrović were cited in the time approximate to that when he wrote them, which is why these citations are not contained in statistics, which are modern, but quite often badly interpreted. Let us mention that the manuscripts published in *Acta Mathematica, Mathematische Annalen* and a number of other journals, which explore problems from his doctoral dissertation, and most other papers published in earlier specified journals in France, were correctly cited in the late 19th and early 20th century. The results of Petrović's first work in *Compte Rendus* were stated in their entirety in the famous, at the time the most valued Picard's monograph *Traité d'analyse*, while the results of his doctoral dissertation were cited in the *Encyclopedia of Mathematics*.

In the vortex of events during the Balkan Wars and World War I, Mihailo Petrović was increasingly less present in France, and thus in world circles within which new science was being developed.

Whether he wanted it or not, Mihailo Petrović was a rather lonely intellectual. He often did not develop his ideas in full, which is why others, inspired by his results and ideas, wrote much more in-depth and more widely cited works. Regardless of Petrović's brilliance and fondness of friendships, he did not have followers to highlight his scientific grandeur. The reason may be the fact that everyone – and here we refer primarily to mathematicians in France, aspired to their personal prestige. Given that there were very few mathematicians in Serbia at the time and that Petrović was our first mathematician who tackled these problems, he could not have an important mathematical support in Serbia, particularly in the period between the two wars. In addition, Mihailo Petrović was not much interested in being cited nor was he interested in having someone to follow him in mathematics, particularly after World War I. On the other hand, mathematicians from the late 19th and early 20th century did not have many options for citation and did not have the habit of such mutual communication in science. In Mihailo Petrović's thesis and works, the citing of results of other mathematicians did not take the form which is acceptable today. The citations in his dissertation are given through the presentation of results ascribed to Picard, Fucs and most certainly to Painlevé. He has in total eight references in his entire doctoral dissertation.

JEDNOTA ČESKOSLOVENSKÝCH MATEMATIKŮ A FYSIKŮ
V PRAZE.

Na valné schůzi dne 20. ledna 1923 byl zvolen

PAN MIHAJLO PETROVIĆ,

ČESTNÝM ČLENEM.

The certificate of membership in the Czechoslovakian Mathematical Society (SASA Archive, 14188/31)

Mihailo Petrović wrote all of his papers alone, apart from one paper which he wrote together with Karamata. This joint work is interesting because the authors corrected Poincaré's error. The reason for this "mathematical loneliness" may be his exceptional individuality, and perhaps, mathematical culture and mathematical knowledge that he possessed, which is why he did not need any associates.

Mihailo Petrović presented his works as a guest lecturer at a number of important international congresses of mathematics in Rome (1908), Cambridge (1912), Toronto (1924), Bologna (1928) and Zurich (1932), and many conferences of scientific federations of France (around ten times), Romania, Italy, Slavic and Balkan countries. He was widely esteemed. He was a member of the Yugoslav Academy of Sciences and Arts, Czech Royal Academy, Polish Academy of Sciences in Krakow, Academy of Sciences in Warsaw, Romanian Academy of Sciences, and many mathematical societies in Paris, Palermo, Bucharest, Leipzig, Prague, Lviv. He was also a member of several other scientific societies in Paris.

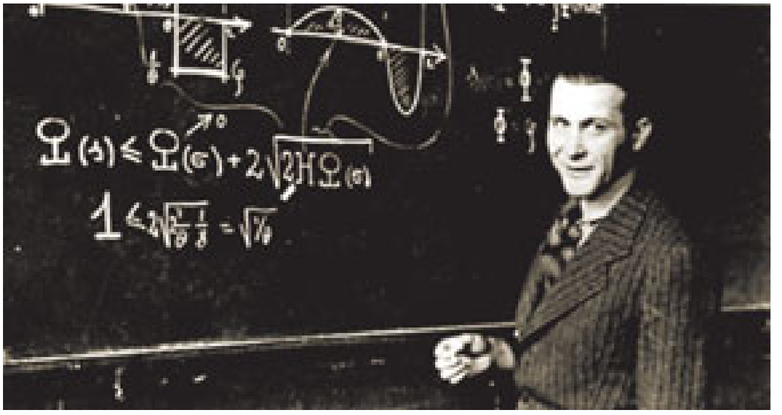
STUDENTS OF MIHAILO PETROVIĆ

Doctoral students of professor Petrović: Tadija Pejović, Konstantin Orlov, Dragoljub Marković (Faculty of Mathematics, University of Belgrade)

Mihailo Petrović came to Serbia with enormous knowledge that he gained in Paris. Learning from the best, he raised the educational system of mathematics to the highest possible level. He understood exceptionally well the classical mathematical fields of analysis and the theory of equations. His lectures are in fact the greatest contribution to the development of mathematics in our country. Petrović passed on to all of his students his excellent knowledge of classical mathematics, primarily of analysis. They continued to develop themselves further in the fields that best fit their preferences and talents, contributing to Serbian and world mathematics. His students, academician Jovan Karamata, professor Dragoslav Mitrinović and professor Tadija Pejović taught the greatest number of doctors of mathematics.

At the time when academician Petrović lived and worked, the most spectacular achievements were made in mathematics. Petrović's contemporaries were Lebesgue, Poincaré, Cartan, Hilbert, Dirac, Smirnov, Fichtenholz and many others. The functional analysis based on Lebesgue's integrals, Hilbert and Banach spaces, including the development of topological and algebraic methods of Poincaré, Hausdorff, Borel, Kolmogorov and many other greats of the time, largely enriched the areas of differential equations.

The majority of Mihailo Petrović's students continued his work in the area of equations, as already explained, and its quality is mainly on a par with Petrović. In addition, some of his followers in the field of analysis studied classical theorems on the mathematical analysis of real and complex functions – particularly the assertions of Abel and Tauber type, with many ingenious mathematical witty remarks, with exceptionally talented Jovan Karamata having a lead role. His proof of Tauber's theorem for the Laplac transform of the non-negative measure is still today largely important in mathematical literature, e.g. microlocal analysis. Karamata himself could not envisage that his theorem would be used nor was he familiar with all the areas in which the theorem is relevant. Of course, some of these areas were studied in his time as well. The works on the theory of analytic functions through professor Tadija Pejović, one of 1300 corporals, and his students, resulted in the creation of the outstanding *Belgrade School of Real and Complex Analysis*, which is still active today. Furthermore, professor Pejović taught distinguished professor Slaviša Prešić who, owing to his great erudition, initiated new mathematical research in mathematical logics and algebra. It is worth noting that Prešić created the so-called *Belgrade School of Logics*, the most famous in post-war Yugoslavia.



Doctoral student of professor Petrović:
Jovan Karamata
(Faculty of Mathematics,
University of Belgrade)

In his article about Mihailo Petrović, academician Tomić wrote that it seemed Petrović had hurried in his scientific work. As if he wanted to compensate for the enormous differences in the scientific level of the country that he came from and world science. It is possible that he therefore did not have time, and perhaps patience either, to fully develop his ideas, which was later used by mathematicians in world centres, who became famous with theories that had no mention of Petrović's ideas. We are referring here to works on differential equations, theory of analytic functions, the already mentioned phenomenology, and works on spectral theory which contain the roots of interval analysis. It seems that the reason for his perhaps rapid neglect of important results that he obtained was the fact that Petrović was lonely, without appropriate conversation partners or associates in the country who could with their questions or in discussions motivate him to go deeper into the analysis of his results. On the other hand, it was Petrović's great creativity that prompted him to quickly overlook some of his ideas. He was simply brimming with ideas.

Academician Petrović did not write papers with his students, but, being mathematically highly educated, he offered to everyone complete classical knowledge which they further used. The Belgrade School of Mathematics is fundamently connected to Petrović and Karamata. Historians of mathematics owe us the analysis of work of the *Belgrade School* before World War II and immediately after the war, in the context of world mathematics, and not in terms of turbulent events in Serbia only. It seems that there is no analysis of the impact of the German occupation. Some events are unreasonably mystified and those less favourable in the historical context of mathematics of the time are disregarded.

What is considered the greatest quality of academician Petrović is his closeness to ordinary people, sympathy with their suffering, as seen in numerous examples from his life. He participated as a Serbian soldier in the Balkan Wars and the Great War. Aged 73, he voluntarily joined the army in World War II. He was captivated and interned in a camp. Upon his return, he passed away in silence and sailed into our history as one of our greatest scientists.

Mihailo Petrović Alas in his late years
(SASA Archive, 14188/21)

PETROVIĆ'S CONTRIBUTION TO HIGHER EDUCATION

In the role assigned to him by a set of circumstances, academician Petrović was exceptionally important for the development of the university teaching of mathematics in Serbia. What we particularly emphasise and what is particularly important to all of us concerns Professor Petrović's mentorship in Serbia. Until World War I and between the two world wars, Petrović was developing the mathematical system to be applied at the universities of Serbia, almost on his own. As of 1894, he was full professor of mathematics at the Great School, and as of 1905 at the University of Belgrade. He was the only professor who in the 1912–1941 period mentored doctoral students in mathematics at the University of Belgrade. He did not publish many textbooks – only three, but his hand-written manuscripts are of exquisite quality and one still derives a lot of satisfaction from them. He held 16 different courses – ten courses on analysis and differential equations, two courses in algebra, three on numerical mathematics, and a separate course on mathematical phenomenology. He wrote eight teaching manuscripts.

He was Member of the Professorial Examination Board, Ministry Supervisor at maturity exams and President of the Main Educational Council of Serbia. He was also an advisor for the preparation of high-school textbooks and wrote several works on methodology.

Mihailo Petrović was a strict and principled professor. Graduate students of mathematics understood this very quickly, paying much more attention to the preparation of exams in mathematical fields. For instance, during one year, Mihailo Petrović held all mathematical subjects at the University of Belgrade. His students and Milutin Milanković later held courses and helped him in teaching. It has been noted that he held not a single public speech, which is not unusual among mathematicians. However, he left behind many interviews and several newspaper articles, describing mathematical life in Serbia.

(SASA Archive, 14197/II-22-1)

Together with Milanković, he launched the journal *Mathematical Publications of Belgrade University*, which later grew into today's *Publications of the Mathematical Institute*. The journal was established in 1932 by academician Anton Bilimović, one of our most famous mechanical engineers, who came from Russia after the October Revolution, and was a close associate of Mihailo Petrović who proposed him for an academician. In addition to the Mathematics Department of the Faculty of Philosophy led by Petrović, the Society of Mathematicians was established by professor Tadija Pejović. All this was very important for the development of mathematics in Serbia. Mihailo Petrović was an unavoidable or, one may say, one of the most important participants in all these activities.

Mathematical offspring of professor Petrović: academician Slobodan Aljančić (Archive of MISASA), academician Bogoljub Stanković (author: Dragan Aćimović, 2016) and professor Slaviša Prešić (author: Dragi Radojević, 2006)

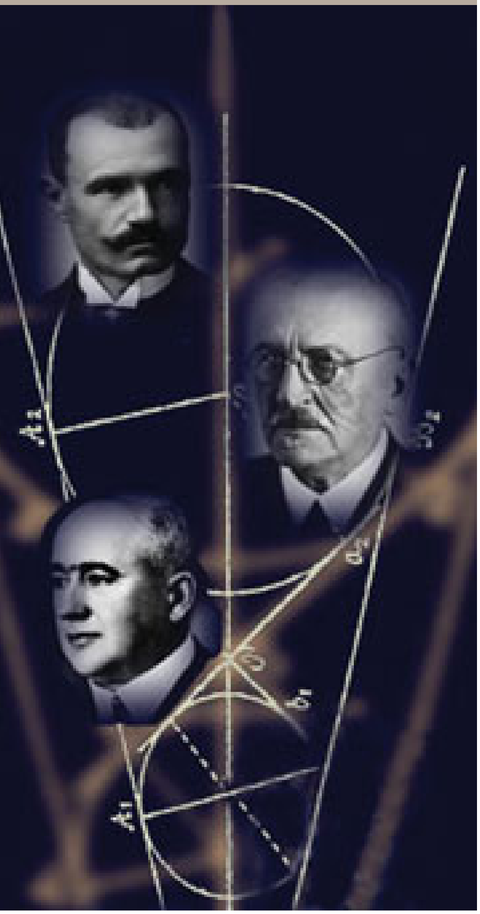
SCIENTIFIC SUCCESSORS OF MIHAILO PETROVIĆ

Academician Mihailo Petrović was to a great extent celebrated by his students, primarily professor Tadija Pejović, academicians Jovan Karamata and Vojislav Avakumović and professor Dragoslav Mitrinović, with a large number of students who, after them, further developed the enormous mathematical genealogical branch of academician Petrović.

The following result worth of great engagement in teaching is unknown for many our mathematicians, mathematical successors of professor Petrović – he is a mathematical predecessor for around 70% of doctors of mathematics in Serbia. The names of academicians, with each following being the mentor of the previous one, appear in the mathematical history of the author of this article: Bogoljub Stanković, Vojislav Avakumović, Jovan Karamata, Mihailo Petrović. In addition to students of academician Đuro Kurepa and mathematicians in Serbia who completed their doctoral studies abroad with different mentors, as well as geometricians whose mathematical roots go to professor Danilo Blanuša, there are many of those whose close mathematical predecessor is Mihailo Petrović.

Let us also mention the names of his students of the first generation: professor Mladen Berić, Sima Marković, Dragoslav Mitrinović, Konstantin Orlov, Tadija Pejović, Dragoljub Marković, Danilo Mihnjević, Petar Muzen, corresponding member of the Serbian Royal Academy Miloš Radojčić, and academicians Radivoj Kašanin and Jovan Karamata; second generation:

academicians Miodrag Tomić, Slobodan Aljančić, Vojislav Marić, Milosav Marjanović, Dragoš Cvetković, Gradimir Milovanović: third generation: Ivan Gutman, Olga Hadžić, Miodrag Mateljević, and the author of this article. I feel a great obligation to mention our exceptionally esteemed mathematicians, successors of academician Petrović, who passed away – professors Manojlo Marović, Ernest Stipanić, Tatomir Anđelić, Milorad Bertolini, Milica and Vojin Dajović, Petar Vasić, Slaviša Prešić, Zagorka Šnajder, Svetozar Milić, Zoran Ivković, Janez Ušan, Dušan Adamović, Dragoljub Aranđelović, Vladeta Vučković, Bogdan Bajšanski, Ranko Bojanić, Tatjana Ostrogorski, Zoran Popstojanović, Ljubomir Protić, Rade Dacić... All of them are related through academician Mihailo Petrović. Many names have not been mentioned. In the genealogy of academician Petrović we can find information about them and about those who are still active and whose names are therefore not mentioned.



Academicians Mihailo Petrović, Bogdan Gavrilović and Milutin Milanković (Faculty of Mathematics, University of Belgrade)

WORK AT THE ACADEMY

Mihailo Petrović Alas became a corresponding member of the Serbian Royal Academy in 1897 and its full member in 1899. The official admission took place in 1900, the same year when Jovan Cvijić became its full member. Petrović was highly active in the Academy's work. He was Secretary of the Department of Natural Sciences and member of the Executive Board of the Academy. He wrote overviews of the works of mathematicians who published in the Academy's *Glas* and also presented new candidates for membership. He published around 60 papers in *Glas*, thus significantly contributing to the Academy's reputation. According to the tradition practiced still today, which is also the tradition of the French Academy, the works of authors outside the Academy were reviewed by members of the Academy. Mihailo Petrović was devoted to these activities particularly because his students, doctoral students and later doctors of mathematics published a great number of their most important works in *Glas* of the Serbian Royal Academy. He promoted a number of academicians – Bogdan Gavrilović, Milutin Milanković, Anton Bilimović and Jovan Karamata.

In 1909, Jovan Cvijić and Mihailo Petrović invited Milutin Milanković to move from Vienna and take a teaching post at the University of Belgrade. In 1920, they proposed him for corresponding member and in 1925 he became a full member of the Academy. When Jovan Cvijić died in 1927, Mihailo Petrović was the most serious candidate for the Academy's president. However, due to the will of the authorities at the time and his friendship with Prince Đorđe Karađorđević, he was not elected. Slobodan Jovanović was elected instead.

Jovan Karamata became a member of the Serbian Royal Academy in 1939. He helped a lot his teacher Mihailo Petrović in preparing scientific publications. Owing to this and the great wish of Mihailo Petrović to bring his papers and legacy in order, we have today a plenty of information about his life and work.

NOVI SAD SCHOOL OF MATHEMATICAL ANALYSIS

Within the Programme Board for Marking the Jubilee, we agreed to prepare texts that connect academician Mihailo Petrović with today's situation of mathematics in Serbia and the areas that we belong to. The text that follows elaborates on the development of mathematical analysis in Novi Sad, functional analysis concerning partial differential equations and harmony analysis through the theory of generalised functions. The genealogical tree, through academicians Karamata and Avakumović, reaches academician Bogoljub Stanković, going to the contemporary research and the scope of mathematical analysis in Novi Sad.

As already stated, academician Jovan Karamata is our most famous and recognised scientist in the world of mathematics among students of academician Petrović. Introducing into mathematics the class of slowly varying functions, he earned a place in the history of world mathematics. This class of functions naturally lies between the class of constants and the class of polynomial functions with equations. It enabled him to achieve a number of brilliant results in the field of function asymptotic in the theorems of Abel and Tauber type for various integral transformations. The leading scientists in classical mathematical analysis in the first quarter of the 20th century, Hardy and Littlewood, were impressed with the elegance of his proof of the Tauber theorem, named after him the *Karamata Tauberian theorem*. Academician Vojislav Avakumović fit into the field developed by academician Karamata. In the already significantly developed theory of regularly varying functions, he defined new classes with appropriate Tauberian results, particularly in the estimate of the number of own values of elliptic operators. Their work was continued by brilliant mathematicians and pedagogues, students of Jovan Karamata – academicians Miodrag Tomić, Slobodan Aljančić and their students at the Faculty of Science and Mathematics. Academician Vojislav Marić used the class of slowly varying functions in the study of solutions of the Thomas-Fermi equation. Also, this class of functions was used by our mathematicians in America, Karamata's students Bojanić, Bajšanski and their successors.

The Novi Sad School of Mathematical Analysis was founded and led by recently deceased academician Bogoljub Stanković. He introduced modern mathematical analysis in Serbia through contemporary areas of functional analysis. Let us also mention that academician

Slobodan Aljančić wrote a high quality textbook on functional analysis at the Faculty of Science and Mathematics in Belgrade.

In the field of mathematical analysis, the 20th century was the period of functional analysis based on algebraic-analytic methods. In its integral part, it contains the theory of generalised functions. The text that follows contains names of important mathematicians, as well as years, in order to better determine the time of their scientific achievements and the development of scientific areas that we elaborate on.

The analysis of functional spaces began in the late 19th century with the works of Ascoli (1843–1896), Volterra (1860–1940), Arzelà (1847–1912), while the Lebesgue integral (in the dissertation from 1902) enabled a qualitatively new approach in all areas of analysis. The functional analysis in the works of Hilbert, Fredholm and Banach, was established in this context, through the synthesis of various areas of geometry, algebra and analysis. Worth mentioning are also Haar (1885–1933), Kolmogorov, Wiener (1894–1946), von Neumann (1903–1957). Particularly important for the development of functional analysis is the development of the modern topology of Borel, Fréchet (1878–1973), Hausdorff (1868–1942) and others.

Within functional analysis, the theory of generalised functions was developed, based on the duality theory. The first results of formal calculus with generalised functions in the solution of differential equations were found with Heaviside (1850–1925). Nobel prize winner Dirac (1902–1984) introduced (~1925) into mathematical physics the calculus with brackets, and Sobolev (1908–1986) introduced (~1930) the concept of the weak derivative in the examination of weak solutions of hyperbolic systems.

L. Schwartz (1920–2003) created (~1950) the theory of distributions, contributing to the development of functional analysis. He also published a monograph which is still today used at post-graduate studies in the theory of linear partial differential equations. Their theories gave an impulse to the theory of pseudo-differential and Fourier integral operators developed by Calderon, Sigmund, and particularly Hörmander, Gelfand, Stein, Bony, and a number of other, mainly European mathematicians. Another approach to the theory of generalised functions, based on the theory of complex functions of several variables and the cohomology theory, was introduced by Sato and his students Kawai and Kashiwara. In this context, we should mention Komatsu, who formulated the theory of ultradistributions (~1970) and Colombo who introduced (~1980) the nonlinear theory of generalised functions, with the aim of studying nonlinear problems.

After his stay in Paris, where he taught as a *maître de conférences* and attended lectures of great mathematicians, Bogoljub Stanković became aware of the importance of functional analysis. This was a period of somewhat older mathematicians than him – Dieudonné, Schwartz and later their students (J. L.) Lions and Grothendieck. Together with Mikusiński, based on algebraic methods in the theory of equations, they developed the theory of *Mikusiński's operator*. An exceptionally important aspect of the activity of academician Stanković was his work with young people, whom he familiarised with modern trends of analysis, with application in solving partial differential equations. The theories of distributions, ultradistributions and hyperfunctions developed in the second half of the 20th century, to which he gave a personal contribution, became a scientific language of an entire group of mathematicians making up the Novi Sad School of Mathematical Analysis, developed through the seminar held almost continuously over the past 60 years on Mondays, at noon at the Novi Sad Faculty of Natural Sciences. Distinguished world mathematicians were guests at the seminar. Students of academician Stanković presented their first scientific results there. In addition, as one of the first associates of the Mathematical Institute, after academician Aljančić, he led the Mathematics Department of the Mathematical Institute. The Novi Sad seminar is thematically primarily devoted to functional analysis, while the seminar of the Mathematical Institute was of a general nature, where our most important mathematicians taught, as well as world-renowned foreign mathematicians, such as Laurent Schwartz.

One of the first students of academician Stanković was academician Olga Hadžić. Her work on the fixed point theory and contribution to leading the journal *NSJOM* and international verification of scientific results of younger associates was exceptionally important.

Academician Stanković's cooperation with eminent world mathematicians Mikusiński, Vladimirov and others resulted in conferences under the general name of *Generalised Functions*. They served as an important impetus first to the structural analysis of various spaces of generalised functions and general integral transforms, and as of recently to application in the theory of partial differential equations and the microlocal analysis with the theory of pseudo-differential and Fourier integral operators. The International Association for Generalised Functions was set up, seated in Vienna, with mathematicians from Novi Sad playing a highly active role in its work. The majority of conferences on generalised functions were held in Novi Sad.

The Novi Sad group applied the strong theorems of functional analysis, particularly the theory of measures and various spaces of generalised functions, to solving equations with singularities to which classical mathematics does not have answers. Particularly important are contacts with scientific groups at universities in Vienna and Torino. Owing to cooperation of the author of this article and one of his first students Marko Nedeljkov (not to mention others) with H. Komatsu, F. Colombo, L. Rodin and M. Oberguggenberger and a number of brilliant mathematicians, new areas became the framework of the scientific work of the Novi Sad group.

Mihailo Petrović and Jovan Karamata

AREAS OF SCIENTIFIC RESEARCH OF THE NOVI SAD SCHOOL OF MATHEMATICS

In addition to the structural results of Avakumović, Marić, Tomić, Aljančić and their Belgrade followers, Karamata's theory of regularly varying functions gained in the works of the Novi Sad School its full meaning through the development of generalised asymptotics and Tauber-type theorems in the spaces of distributions, ultradistributions and hyperfunctions. The microlocal analysis of pseudo-differential and Fourier integral operators, including applications in the analysis of the spread of singularities through the wavefront, hypoellipticity and time-frequency analysis, are the dominant research topics. The framework and wavelet theory gives a clear perspective of applications in the analysis of signals. Research areas are maintenance laws and singular solutions of the so-called gradient catastrophe, with solutions containing distributions, as well as the fluid dynamics. Furthermore, the equations of evolution with the asymptotic of hypercyclical and chaotic orbits of semigroups, *H*-measures or microlocal defect measures and their natural generalisations of *H*-distributions and ultradistributions describe, for instance, the homogenisation of particular structures. The stochastic analysis and stochastic equations with uncontrolled noise (the so-called white noise), in initial conditions, i.e. stochastic perturbations of the ideal situation in a particular environment, are studied through stochastic differential equations within the Malliavin calculus and chaos expansions. Fractional differential equations with applications in the examination of models with viscoelastic materials, with the use of the distribution theory method, are also an important research area within the *Novi Sad School of Analysis.*