ISSN 0561-7332

ACADEMIE SERBE DES SCIENCES ET DES ARTS

# BULLETIN

## TOME CLI

## CLASSE DES SCIENCES MATHEMATIQUES ET NATURELLES

## SCIENCES MATHEMATIQUES

N° 43

BEOGRAD 2018

# BULLETIN

## TOME CLI

CLASSE DES SCIENCES
MATHEMATIQUES ET NATURELLES

SCIENCES MATHEMATIQUES

N° 43

ISSN 0561-7332

ACADEMIE SERBE DES SCIENCES ET DES ARTS

# BULLETIN

## TOME CLI

CLASSE DES SCIENCES
MATHEMATIQUES ET NATURELLES

SCIENCES MATHEMATIQUES

N° 43

Rédacteur

GRADIMIR V. MILOVANOVIĆ
Membre de l'Académie

BEOGRAD 2018

Publie et impimé par
*Académie serbe des sciences et des arts*
Beograd, Knez Mihailova 35

Tirage 300 exemplaires

TABLE DES MATIÈRES

*Bulletin T. CLI de l'Académie serbe des sciences et des arts − 2018*
*Classe des Sciences mathématiques et naturelles*
*Sciences mathématiques, № 43*

# NOTE ON IRREGULAR GRAPHS

IVAN GUTMAN, TAMÁS RÉTI

*Dedicated to Professor Bogoljub Stanković (1924–2018)*

(Presented at the 1st Meeting, held on February 23, 2018)

A b s t r a c t. *Let $G$ be a graph with vertex set $\mathbf{V}(G)$ and edge set $\mathbf{E}(G)$. For $v \in \mathbf{V}(G)$, by $d_G(v)$ is denoted the degree of the vertex $v$. A graph in which not all vertices have equal degrees is said to be irregular. Different quantitative measures of irregularity have been proposed, of which the Albertson index $irr(G) = \sum_{uv\in\mathbf{E}(G)} |d_G(u) - d_G(v)|$ is the most popular. We compare $irr(G)$ with the recently introduced sigma-index $\sigma(G) = \sum_{uv\in\mathbf{E}(G)} [d_G(u) - d_G(v)]^2$ and show that in the general case these are incomparable. Graphs in which $|d_G(u) - d_G(v)| = 1$ holds for all $uv \in \mathbf{E}(G)$ are called stepwise irregular (SI). Several methods for constructing SI graphs are described.*

AMS Mathematics Subject Classification (2000): 05C07, 05C35.
Key Words: degree (of vertex), irregularity (of graph), stepwise irregular graph, Albertson index, $\sigma$ index.

## 1. *Introduction*

Let $G$ be a simple undirected connected graph with vertex set $\mathbf{V}(G)$ and edge set $\mathbf{E}(G)$, Let $|\mathbf{V}(G)| = n$ and $|\mathbf{E}(G)| = m$, in which case $G$ is said to be an $(n, m)$-graph. The degree of a vertex $v \in \mathbf{V}(G)$ is the number of edges incident with $v$ and it is denoted by $d_G(v)$. A vertex of degree one is said to be pendent. An edge incident to a pendent vertex is said to be also pendent.

A graph $G$ is regular if all its vertices have the same degree, otherwise it is irregular. In many applications and problems it is of importance to know how irregular a given graph is, i.e., to have a quantitative measure of graph irregularity [16, 42].

There have been several attempts to determine how irregular a graph is [16, 13, 10, 11, 12, 17, 35], but these have not been captured by a single parameter. It seems that the oldest numerical measure of graph irregularity was proposed by Collatz and Sinogowitz [19] and was defined as

$$\lambda_1 - \frac{2m}{n},$$

where $\lambda_1$ is the largest eigenvalue of the adjacency matrix, usually referred to as the spectral radius of the underlying graph [20, 44]. Recall that the spectral radius of a simple $(n,m)$-graph satisfies $\max\{\overline{d}, \sqrt{\Delta}\} \leq \lambda_1 \leq \Delta$, where $\Delta$ is the maximal vertex degree and $\overline{d}(G) = 2m/n$ the average degree of the graph $G$. It follows, that if a graph is regular, then $\lambda_1 = 2m/n$ and $\lambda_1 > 2m/n$ otherwise.

A somewhat more straightforward measure of irregularity was put forward by Bell [14], who proposed that the variance $\mathrm{Var}(G)$ of the vertex degrees

$$\mathrm{Var}(G) = \frac{1}{n} \sum_{v \in \mathbf{V}(G)} d_G(v)^2 - \left( \frac{1}{n} \sum_{v \in \mathbf{V}(G)} d_G(v) \right)^2 . \tag{1.1}$$

serves for this purpose.

It can be easily seen that the Bell irregularity index, Eq. (1.1), can be written as

$$\mathrm{Var}(G) = \frac{1}{n} M_1(G) - \left( \frac{2m}{n} \right)^2 ,$$

where $M_1(G)$ is the classical first Zagreb index [29, 40, 15], defined as

$$M_1(G) = \sum_{v \in \mathbf{V}(G)} d_G(v)^2 = \sum_{uv \in \mathbf{E}(G)} \left[ d_G(u) + d_G(v) \right] .$$

Several additional measures of graph irregularity were proposed [24, 38, 43], but until now these have not attracted much attention.

The imbalance of an edge $e = uv \in \mathbf{E}(G)$, defined as

$$imb(e) = |d_G(u) - d_G(v)| ,$$

appears implicitly in a paper by Albertson and Berman [9], in the context of Ramsey problems, and later in [18]. In [8], Albertson defined the irregularity of $G$ as the sum of imbalances of all edges, i.e.,

$$irr(G) = \sum_{e \in \mathbf{E}(G)} imb(e) = \sum_{uv \in \mathbf{E}(G)} |d_G(u) - d_G(v)| \,. \tag{1.2}$$

Eventually, the Albertson index $irr$ became the most popular and most thoroughly investigated irregularity measure, see [2, 3, 32, 30, 30, 33, 27, 34, 45, 25, 21, 36, 37, 39, 46]. Recently, a variant of the Albertson index, named "total irregularity", was considered [47, 7, 1, 41, 4, 5, 6, 22, 23], defined as

$$\sum_{\{u,v\} \subseteq \mathbf{V}(G)} |d_G(u) - d_G(v)| \,.$$

Trying to avoid the absolute value calculation in the Albertson index, Eq. (1.2), one naturally arrived at the irregularity index $\sigma(G)$, defined as [31]

$$\sigma(G) = \sum_{uv \in E(G)} \big[d_G(u) - d_G(v)\big]^2 \,. \tag{1.3}$$

In contrast to the Albertson index, the $\sigma$-index can be expressed in terms of the earlier much studied second Zagreb index $M_2(G)$ [40, 15] and the forgotten index $F(G)$ [26], namely

$$\sigma(G) = F(G) - 2\, M_2(G),$$

where

$$F(G) = \sum_{v \in \mathbf{V}(G)} d_G(v)^3 = \sum_{uv \in \mathbf{E}(G)} \big[d_G(u)^2 + d_G(v)^2\big]$$

and

$$M_2(G) = \sum_{uv \in \mathbf{E}(G)} d_G(u)\, d_G(v) \,.$$

Because of the close similarity between the Albertson index, Eq. (1.2) and the $\sigma$-index, Eq. (1.3), it is reasonable to expect that they are consistent with regard to measuring graph irregularity. Thus, it could be expected that the condition

$$irr(G_1) \geq irr(G_2) \;\Leftrightarrow\; \sigma(G_1) \geq \sigma(G_2) \tag{1.4}$$

is satisfied by any pair of graphs $G_1, G_2$.

In what follows, we point out that relation (1.4) is not generally valid. This casts serious doubts on the true meaning of what one refers to as "irregularity", and the attempts to measure it by means of a single parameter.

If condition (1.4) would not hold for the graphs $G_1$ and $G_2$, then the two measures $irr$ and $\sigma$ would infer different orderings of their irregularity. If so, then the irregularity measures $irr$ and $\sigma$ would be inconsistent with regard to $G_1$ and $G_2$, i.e., would be mutually inconsistent in the general case.

## 2. *Inconsistence between Albertson and $\boldsymbol{\sigma}$-Index*

In this section we consider the inconsistency cases for the Albertson and $\sigma$-indices, i.e., seek for pairs of graphs for which

$$irr(G_1) > irr(G_2) \qquad \text{and} \qquad \sigma(G_1) < \sigma(G_2)$$

holds. This condition can be rewritten in the form:

$$\big[irr(G_1) - irr(G_2)\big]\big[\sigma(G_1) - \sigma(G_2)\big] < 0\,. \tag{2.1}$$

We first focus our attention to trees, that is connected $(n, n-1)$-graphs.

In order to find a pair of $n$-vertex trees satisfying inequality (2.1), let $T_1(n)$ be a tree without vertices of degree greater than 3, with Albertson index as large as possible. Such a tree possesses only vertices of degree three and pendent vertices. Their numbers are denoted by $n_3$ and $n_1$, respectively.

It is easy to see that the imbalance of edges connecting two degree–three vertices is zero, and the imbalance of the pendent edges is 2. Therefore, $irr(T_1(n)) = 2\,n_1$ and $\sigma(T_1(n)) = 2^2\,n_1$.

Because of $n_1 + n_3 = n$ and $n_1 + 3\,n_3 = 2(n-1)$, we get $n_1 = n/2 - 1$. Thus, the tree $T_1(n)$ must possess an even number of vertices, and

$$irr(T_1(n)) = n + 2 \qquad \text{and} \qquad \sigma(T_1(n)) = 2n + 4\,. \tag{2.2}$$

Let $T_2(n)$ be an $n$-vertex tree with vertices of degree greater than 3, with Albertson index as small as possible. This tree must possess a single vertex of degree 4, four pendent vertices, and $n-5$ vertices of degree 2. Denote by $p_4$ the number of pendent vertices attached to the degree 4 vertex of $T_2(n)$. It is easy to verify that irrespective of the actual value of the parameter $n$,

$$irr(T_2(n)) = \begin{cases} 4\times 2 + 4\times 1 + 0\times 3 = 12 & \text{if } p_4 = 0 \\ 3\times 2 + 3\times 1 + 1\times 3 = 12 & \text{if } p_4 = 1 \\ 2\times 2 + 2\times 1 + 2\times 3 = 12 & \text{if } p_4 = 2 \\ 1\times 2 + 1\times 1 + 3\times 3 = 12 & \text{if } p_4 = 3 \end{cases} \tag{2.3}$$

and

$$\sigma(T_2(n)) = \begin{cases} 4\times 2^2 + 4\times 1^2 + 0\times 2^2 = 20 & \text{if } p_4 = 0 \\ 3\times 2^2 + 3\times 1^2 + 1\times 3^2 = 24 & \text{if } p_4 = 1 \\ 2\times 2^2 + 2\times 1^2 + 2\times 3^2 = 26 & \text{if } p_4 = 2 \\ 1\times 2^2 + 1\times 1^2 + 3\times 3^2 = 32 & \text{if } p_4 = 3 \end{cases} \tag{2.4}$$

Substituting Eqs. (2.2)–(2.4) back into (2.1), we get

$$\big[irr(T_1(n) - irr(T_2(n))\big]\big[\sigma(T_1(n)) - \sigma(T_2(n))\big]$$

$$= \begin{cases} 2(n-10)(n-8) & \text{if } p_4 = 0 \\ 2(n-10)(n-10) & \text{if } p_4 = 1 \\ 2(n-10)(n-12) & \text{if } p_4 = 2 \\ 2(n-10)(n-14) & \text{if } p_4 = 3\,. \end{cases}$$

Bearing in mind that $n$ must be an even integer, we see that the condition (2.1) can be satisfied only if $p_4 = 3$ and only for $n = 12$.

In Fig. 1 is depicted a tree $T_1(12)$ (of the two possible), as well as the unique tree $T_2(12)$ with $p_4 = 3$. These are the smallest possible examples of trees for which the Albertson and $\sigma$ indices are inconsistent.

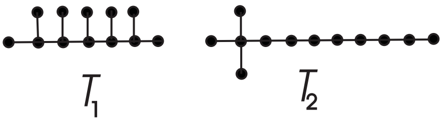


**Figure 1.** Two 12-vertex trees inconsistent with regard to Albertson and $\sigma$-index: $irr(T_1) = 14$, $irr(T_2) = 12$ whereas $\sigma(T_1) = 28$, $\sigma(T_2) = 32$.

In an analogous manner we may construct pairs of connected unicyclic, bicyclic, and higher–cyclic graphs for which the Albertson and $\sigma$ indices are inconsistent. The species depicted in Fig. 2 are the smallest possible of their kind.

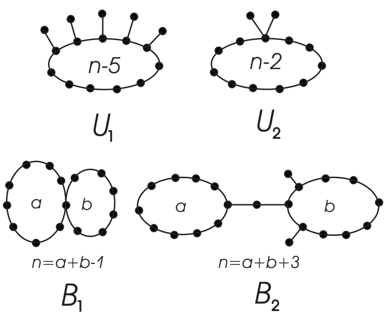


**Figure 2.** Unicyclic and bicyclic graphs inconsistent with regard to Albertson and $\sigma$-index: $irr(U_1) = 12$, $irr(U_2) = 10$ whereas $\sigma(U_1) = 22$, $\sigma(U_2) = 26$; $irr(B_1) = 8$, $irr(B_2) = 10$ whereas $\sigma(B_1) = 16$, $\sigma(B_2) = 14$. These values hold for any feasible choices of the parameters $a$, $b$ and $n$.

## 3. *Stepwise irregular graphs*

A graph $G$ in which the imbalance $imb(e)$ of any edge $e \in \mathbf{E}(G)$ is unity, is referred to as a stepwise irregular (SI) graph [28]. Evidently, among graphs with non-zero imbalance, the stepwise irregular species have minimal irregularity. In the recent paper [28], it was shown how SI graphs can be systematically constructed, provided that a single SI graph with pendent vertices is known. This we refer to as:

*Construction method 1* [*28*]

Let $v$ be a pendent vertex of the graph $G_1$, adjacent to the vertex $u$, see Fig. 3. If the graph $G_1$ is stepwise irregular (in which case the degree of the vertex $u$ must be 2), then also the graph $G_1^*$ is stepwise irregular. Since $G_1^*$ possesses a pendent vertex, adjacent to a vertex of degree 2, the construction of SI graphs can be continued.

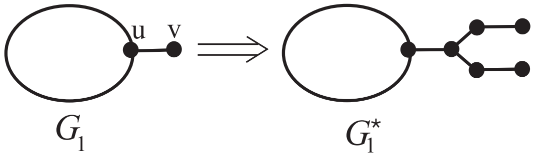


**Figure 3.** Construction of SI graphs according to the method from the paper [28]

In this section, we point out a few additional general methods for constructing SI graphs.

*Construction method 2*

Let $v$ be a vertex of the graph $G_2$ having degree 2, adjacent to the vertices $u_1$ and $u_2$, see Fig. 4. If the graph $G_2$ is stepwise irregular so that the vertices $u_1$ and $u_2$ are both of degree 3, then also the graph $G_2^*$ is stepwise irregular. Since $G_2^*$ possesses a vertex of degree 2, adjacent to two vertices of degree 3, the construction of SI graphs can be continued.

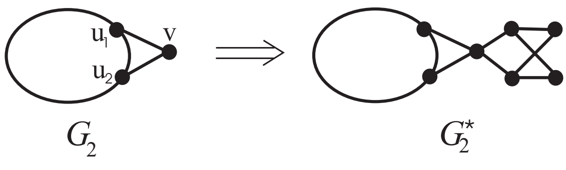


**Figure 4.** Construction of SI graphs according to the method 2

*Construction method 3*

Let $v$ be a vertex of the graph $G_3$ having degree 3, adjacent to the vertices $u_1$, $u_2$, and $u_3$, see Fig. 5. If the graph $G_3$ is stepwise irregular so that the vertices $u_1$, $u_2$, and $u_3$ are all of degree 4, then also the graph $G_3^*$ is stepwise irregular. Since $G_3^*$ possesses a vertex of degree 3, adjacent to three vertices of degree 4, the construction of SI graphs can be continued.

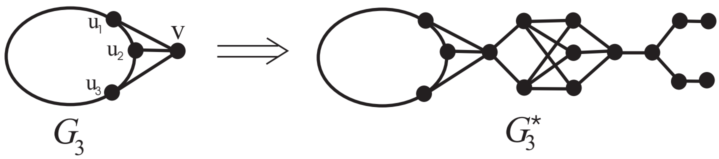


**Figure 5.** Construction of SI graphs according to the method 3

A connected graph is said to be bidegreed if some of its vertices are of degree $\Delta$ and the other vertices are of degree $\delta$, $\Delta > \delta > 0$. A bidegreed bipartite graph is called semiregular if each vertex in the same part of bipartition has the same degree.

*Construction method 4*

Denote by $K_{p,q}$ the complete bipartite graph with $n = p+q$ vertices and $m = pq$ edges. Complete bipartite graphs with $p \neq q$ form a subset of semiregular graphs. If $q = p+1$, then the graphs $K_{p,p+1}$ are semiregular and stepwise irregular. It is easy to see that the 3-vertex star $K_{1,2}$ and the 5-vertex bicyclic graph $K_{2,3}$ are the smallest stepwise irregular graphs. Both of them are semiregular.

Recall that the cyclomatic number of a connected graph $G$ with $n$ vertices and $m$ edges is $\gamma(G) = m - n + 1$. A cubic graph is a regular graph of degree 3. A cubic graph of order $n$ has $m = \frac{3}{2}n$ edges, i.e., it is an $(n, \frac{3}{2}n)$-graph.

*Construction method 5*

Stepwise irregular graphs can be constructed from cubic graphs or cubic multigraphs. Let $G$ be a connected (multi)graph, and denote by $S(G)$ its subdivision graph. If $G$ is a cubic regular $(n, \frac{3}{2}n)$-graph, with cyclomatic number $\gamma(G)$, then $S(G)$ will be a bidegreed semiregular stepwise irregular $(\frac{5}{2}n, 3n)$-graph with degree set $\{2, 3\}$, having the same cyclomatic number as $G$.

In Fig. 6 is depicted a sequence of small bidegreed and tridegreed SI graphs, denoted by $H_n$, where the subscript $n$ stands for their order.

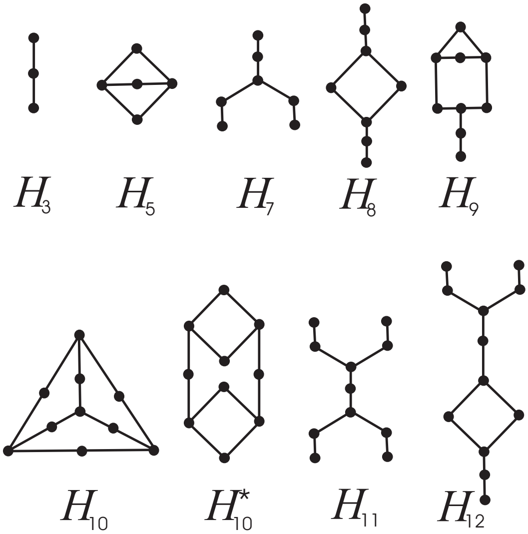


**Figure 6.** Stepwise irregular graphs with small cyclomatic numbers

The cyclomatic number of the graphs $H_n$ is small, $\gamma = 0, 1, 2$, or 3. Among them, $H_3 \cong K_{1,2}$, $H_7$, and $H_{11}$ are trees, $H_8$ and $H_{12}$ are unicyclic graphs, $H_5 \cong K_{2,3}$ and $H_9$ are bicyclic graphs, whereas $H_{10}$ and $H^*_{10}$ are tricyclic. The graphs $H_3$, $H_5$, $H_{10}$, and $H^*_{10}$ are semiregular. The graphs $H_5$ and $H^*_{10}$ are subdivision graphs of the cubic multigraphs $J_2$ and $J^*_4$, depicted in Fig. 7, whereas $H_{10}$ is the subdivision graph of the complete graph $K_4$ (which is regular of degree 3).

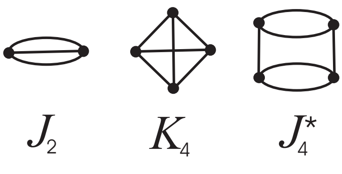


**Figure 7.** Cubic graphs from which the SI graphs $H_5$, $H_{10}$, and $H^*_{10}$ are constructed

In the recent paper [28], the following two results were stated:

**Theorem 10.** [28] *There exist stepwise irregular bicyclic graphs whose order is any positive odd integer, except* 1*,* 3*,* 5*,* 7*,* 9*, and* 11.

**Theorem 11.** [28] *There exist connected stepwise irregular graphs of any order, except* 1*,* 2*,* 4*,* 5*, and* 6.

Considering the SI graphs shown in Fig. 7, and bearing in mind the graphs $H_5$ and $H_9$, the following minor correction to these theorems need to be made:

**Theorem 10** (corrected). *There exist stepwise irregular bicyclic graphs whose order is any positive odd integer, except* 1*,* 3*,* 7*, and* 11.

**Theorem 11** (corrected). *There exist connected stepwise irregular graphs of any order, except* 1*,* 2*,* 4*, and* 6.

REFERENCES

[1] H. Abdo, S. Brandt, D. Dimitrov, *The total irregularity of a graph*, Discrete Math. Theor. Comput. Sci. **16** (2014), 201–206.

[2] H. Abdo, N. Cohen, D. Dimitrov, *Graphs with maximal irregularity*, Filomat **28** (2014), 1315–1322.

[3] H. Abdo, D. Dimitrov, *The irregularity of graphs under graph operations*, Discuss. Math. Graph Theory **34** (2014), 263–278.

[4] H. Abdo, D. Dimitrov, *The total irregularity of graphs under graph operations*, Miskolc Math. Notes **15** (2014), 3–17.

[5] H. Abdo, D. Dimitrov, *The total irregularity of some composite graphs*, Int. J. Comput. Appl. **122** (2015), 1–9.

[6] H. Abdo, D. Dimitrov, *Non-regular graphs with minimal total irregularity*, Bull. Austal. Math. Soc. **92** (2015), 1–10.

[7] H. Abdo, D. Dimitrov, W. Gao, *On the irregularity of some molecular structures*, Canad. J. Chem. **95** (2017), 174–183.

[8] M. O. Albertson, *The irregularity of a graph*, Ars Comb. **46** (1997), 219–225.

[9] M. O. Albertson, D. Berman, *Ramsey graphs without repeated degrees*, Congr. Numer. **83** (1991), 91–96.

[10] Y. Alavi, A. Boals, G. Chartrand, P. Erdős, O. R. Oellermann, *$k$-path irregular graphs*, Congr. Numer. **65** (1988), 201–210.

[11] Y. Alavi, G. Chartrand, F. R. K. Chung, P. Erdős, R. L. Graham, O. R. Oellermann, *Highly irregular graphs*, J. Graph Theory **11** (1987), 235–249.

[12] Y. Alavi, J. Liu, J. Wang, *Highly irregular digraphs*, Discrete Math. **111** (1993), 3–10.

[13] F. K. Bell, *On the maximal index of connected graphs*, Linear Algebra Appl. **144** (1991), 135–151.

[14] F. K. Bell, *A note on the irregularity of graphs*, Linear Algebra Appl. **161** (1992), 45–54.

[15] B. Borovićanin, K. C. Das, B. Furtula, I. Gutman, *Bounds for Zagreb indices*, MATCH Commun. Math. Comput. Chem. **78** (2017), 17–100.

[16] G. Chartrand, P. Erdős, O. R. Oellermann, *How to define an irregular graph*, College Math. J. **19** (1988), 36–42.

[17] G. Chartrand, K. S. Holbert, O. R. Oellermann, H. C. Swart, *$F$-degrees in graphs*, Ars Comb. **24** (1987), 133–148.

[18] G. Chen, P. Erdős, C Rousseau, R. Schelp, *Ramsey problems involving degrees in edge-colored complete graphs of vertices belonging to monochromatic subgraphs*, Eur. J. Comb. **14** (1993), 183–189.

[19] L. Collatz, U. Sinogowitz, *Spektren endlicher Graphen*, Abh. Math. Sem. Univ. Hamburg **21** (1957), 63–77.

[20] D. Cvetković, P. Rowlinson, S. Simić, *An Introduction to the Theory of Graph Spectra*, Cambridge Univ. Press, Cambridge, 2010.

[21] N. De, A. Pal, S. M. Abu Nayeem, *The irregularity of some composite graphs*, Int. J. Appl. Comput. Math **2** (2016), 411–420.

[22] D. Dimitrov, T. Réti, *Graphs with equal irregularity indices*, Acta Polytechn. Hung. **11**(4) (2014), 41–57.

[23] D. Dimitrov, R. Škrekovski, *Comparing the irregularity and the total irregularity of graphs*, Ars Math. Contemp. **9** (2015), 25–30.

[24] C. Elphick, P. Wocjan, *New measures of graph irregularity*, El. J. Graph Theory Appl. **2**(1) (2014), 52–65.

[25] G. H. Fath–Tabar, I. Gutman, R. Nasiri, *Extremely irregular trees*, Bull. Cl. Sci. Math. Nat. Sci., Math. **145** (2013), 1–8.

[26] B. Furtula, I. Gutman, *A forgotten topological index*, J. Math. Chem. **53** (2015), 1184–1190.

[27] I. Gutman, *Irregularity of molecular graphs*, Kragujevac J. Sci. **38** (2016), 71–81.

[28] I. Gutman, *Stepwise irregular graphs*, Appl. Math. Comput. **325** (2018), 234–238.

[29] I. Gutman, K. C. Das, *The first Zagreb index 30 years after*, MATCH Commun. Math. Comput. Chem. **50** (2004), 83–92.

[30] I. Gutman, P. Hansen, H. Mélot, *Variable neighborhood search for extremal graphs. 10. Comparison of irregularity indices for chemical trees*, J. Chem. Inf. Model. **45** (2005), 222–230.

[31] I. Gutman, M. Togan, A. Yurttas, A. S. Cevik, I. N. Cangul,*Inverse problem for sigma index*, MATCH Commun. Math. Comput. Chem. **79** (2018), 491–508.

[32] P. Hansen, H. Mélot, *Variable neighborhood search for extremal graphs. 9. Bounding the irregularity of a graph*, Discrete Math. Theoret. Comput. Sci. **69** (2005), 253–264.

[33] M. A. Henning, D. Rautenbach, *On the irregularity of bipartite graphs*, Discrete Math. **307** (2007), 1467–1472.

[34] B. Horoldagva, L. Buyantogtokh, S. Dorjsembe, I. Gutman, *Maximum size of maximally irregular graphs*, MATCH Commun. Math. Comput. Chem. **76** (2016) 81–98.

[35] D. E. Jackson, R. Entringer, *Totally segregated graphs*, Congr. Numer. **55** (1986), 159–165.

[36] F. Liu, Z. Zhang, J. Meng, *The size of maximally irregular graphs and maximally irregular triangle–free graphs*, Graphs Comb. **30** (2014), 699–705.

[37] W. Luo, B. Zhou, *On the irregularity of trees and unicyclic graphs with given matching number*, Util. Math. **83** (2010) 141–148.

[38] S. Mukwembi, *On maximally irregular graphs*, Bull. Malays. Math. Sci. Soc. **36** (2013), 717–721.

[39] R. Nasiri, G. H. Fath–Tabar, *The second minimum of the irregularity of graphs*, El. Notes Discr. Math. **45** (2014), 133–140.

[40] S. Nikolić, G. Kovačević, A. Miličević, N. Trinajstić, *The Zagreb indices 30 years after*, Croat. Chem. Acta **76** (2003) 113–124.

[41] T. Réti, D. Dimitrov, *On irregularities of bidegreed graphs*, Acta Polytechn. Hung. **10**(7) (2013), 117–134.

[42] T. Réti, R. Sharafdini, A. Dregélyi–Kiss, H. Haghbin, *Graph irregularity indices used as molecular descriptors in QSPR studies*, MATCH Commun. Math. Comput. Chem. **79** (2018), 509–524.

[43] T. Réti, E. Tóth–Laufer, *On the construction and comparison of graph irregularity indices*, Kragujevac J. Sci. **39** (2017), 53–75.

[44] D. Stevanović, *Spectral Radius of Graphs*, Elsevier, Amsterdam, 2015.

[45] M. Tavakoli, F. Rahbarnia, M. Mirzavaziri, A. R. Ashrafi, I. Gutman, *Extremely irregular graphs*, Kragujevac J. Math. **37** (2013), 135–139.

[46] B. Zhou. W. Luo, *On irregularity of graphs*, Ars Comb. **88** (2008), 55–64.

[47] Y. Zhu, L. You, J. Yang, *The minimal total irregularity of some classes of graphs*, Filomat **30** (2016), 1203–1211.

Faculty of Science
University of Kragujevac
P. O. Box 60
34000 Kragujevac
Serbia
e-mail: gutman@kg.ac.rs

Donát Bánki Faculty of Mechanical and Safety Engineering
Óbuda University
H-1081 Budapest
Hungary
e-mail: reti.tamas@bgk.uni-obuda.hu