



ACADEMIE SERBE DES SCIENCES ET DES ARTS

BULLETIN

TOME CLIV

CLASSE DES SCIENCES
MATHEMATIQUES ET NATURELLES

SCIENCES MATHÉMATIQUES

N° 46

BULLETIN T. CLIV – SCIENCES MATHÉMATIQUES N° 46

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Rédacteur

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BEOGRAD 2021

Publie et imprimé par
Académie serbe des sciences et des arts
Beograd, Knez Mihailova 35

Tirage 300 exemplaires

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IN MEMORIAM

ALEKSANDAR IVIĆ (MARCH 6, 1949 – DECEMBER 27, 2020)



Eminent scientist Aleksandar Sanja Ivić, Full Member of the Serbian Academy of Sciences and Arts (SANU), passed away on December 27, 2020, after a vicious illness he faced with an unprecedented will, but unfortunately he could not cope.



Figure 1: Aleksandar with his parents Milka and Pavle

He was born on March 6, 1949 in Belgrade. The son of highly esteemed Academicians Milka and Pavle Ivića (Fig. 1) he inherited exceptional intelligence and love for books from his parents, but his talent for mathematics is something that especially set him apart from his famous parents and famous ancestors, including his grandfather (by father) historian Aleksa Ivić, and great-grand-father (by mother), a poet Vojislav Ilić, who was the of son of a writer and minister Jovan Ilića. Aleksandar Sanja Ivić gained his fame, his reputation, greater in the world than in Serbia, as a mathematical genius.

Sanja Ivić attended “Đorđe Natošević” Elementary School and “Jovan Jovanović Zmaj” Grammar School in Novi Sad. He had always been the best, and his peers used to say that Sanja had already excelled in all forms of education, but that it was good that sports such as football, chess and others existed, where other boys could excel too. He showed a talent for mathematics very early. He participated in various competitions in mathematics in Serbia and ex-Yugoslavia, as well as at the Mathematical Olympiad and almost always won at them. He won TV quiz shows, where he solved chess problems. He also played classical guitar and had concerts (see Fig. 2).

Sanja Ivić completed his mathematics studies at the University of Novi Sad with an average mark 10 (ten). During his high school education and studies, he stayed in America, where he attended courses with the world’s leading mathematicians at the time, especially in the fields of complex analysis. The biggest unsolved mathemati-



Figure 2: Concert at Faculty of Medicine (Novi Sad, April 28, 1972)

cal problem today, known as the Riemann Hypothesis, was a roadmap to Analytical Number Theory.

Aleksandar Ivić continued his postgraduate studies in the Mathematics group at the Faculty of Natural Sciences and Mathematics, University of Belgrade, where he received his Master's Degree in 1973. He defended his Doctoral Dissertation entitled "*On some classes of arithmetic functions related to the distribution of prime numbers*" in 1975 at the same faculty. The preparation of the Master's and Doctoral Theses was supervised by the well-known Yugoslav mathematician, Academician Đuro Kurepa (1907–1993).

From 1971 to 1976 (except for the time 1973–74 spent in the Yugoslav Army), Aleksandar Ivić was an Assistant Mathematician at the Faculty of Science in Novi Sad. In September 1976, he transferred to the Faculty of Mining and Geology, University of Belgrade (Department of Applied Mathematics), passing through all university titles. He was elected Full Professor in 1988, and Professor Emeritus at the University of Belgrade in 2014, when he formally retired.

Academician Aleksandar Ivić was elected Corresponding Member of the Serbian Academy of Sciences and Arts (SASA) in 1988, and became its Full Member in 2000. Until recently, he was the President of the Finance Committee at SASA, leaving the vacant position of Head of the Mathematics Department within the Department of Mathematics, Physics and Geo-Sciences. He was also a Member of the National Council for Higher Education of Serbia. Ivić was also a talent for languages. He spoke excellent English, French, German, Russian, Italian and Spanish.

Academician Aleksandar Ivić's scientific work had been ongoing since 1973. He primarily studied mathematical problems in Number Theory, especially in Analytical Number Theory, where various methods of mathematical analysis are used to approach problems. They include complex analysis, Laplace, Fourier and Mellin integral transformation, as well as methods of classical and functional analysis. Ivić's main results concern arithmetic functions (summation formulas, value distribution, large and small values), Riemann zeta-function(s) and related series (density theorems, moments, value distribution), as well as problems divisors (the classic Dirichlet problem of divisors with generalizations, as well as divisor problems related to various products of the zeta-function degree). His papers and books have been noted for their quality and published in famous journals such as "*Inventiones Mathematicae*"; "*Mathematische Annalen*" and "*Journal of the London Mathematical Society*". Moreover, his papers have been noted for the quantity (over 280 papers, most of which he wrote himself), for their length as well as for citations (several thousand times). Although he wrote most of his papers by himself, he wrote over 50 joint papers with prominent foreign mathematicians, among which M. Jutila, Y. Motohashi, C. Pomerance, J.-M. De Koninck, G. Tenenbaum, etc. It is worth mentioning that he wrote eight joint works with the famous Hungarian mathematician Paul Erdős (1913–1996).

In addition to these papers, Academician Ivić published several papers in other mathematical and applied fields (integral equations, mathematical chemistry, solid state physics, electrochemistry), thus clearly demonstrating the ability to act outside a narrower field. Academician Aleksandar Ivić published five monographs with well-known world publishers:

- [1] J.-M. De Koninck, **A. Ivić**, *Topics in arithmetical functions. Asymptotic formulae for sums of reciprocals of arithmetical functions and related results*. Notas de Matemática [Mathematical Notes], 72. North-Holland Publishing Co., Amsterdam-New York, 1980. xvii+262 pp. ISBN: 0-444-86049-5.
- [2] **A. Ivić**, *Topics in recent zeta function theory*. Publications Mathématiques d'Orsay [Mathematical Publications of Orsay], 83-6. Université de Paris-Sud, Département de Mathématiques, Orsay, 1983. iii+272 pp.
- [3] **A. Ivić**, *The Riemann zeta-function. The theory of the Riemann zeta-function with applications*. A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1985. xvi+517 pp. ISBN: 0-471-80634-X [Second edition: Dover Publications, Inc., Mineola, NY, 2003. xxii+517 pp. ISBN: 0-486-42813-3].
- [4] **A. Ivić**, *Lectures on mean values of the Riemann zeta function*. Tata Institute of Fundamental Research Lectures on Mathematics and Physics, 82. Published for the Tata Institute of Fundamental Research, Bombay; by Springer-Verlag, Berlin, 1991. viii+363 pp. ISBN: 3-540-54748-7.
- [5] **A. Ivić**, *The theory of Hardy's Z-function*. Cambridge Tracts in Mathematics,

196. Cambridge University Press, Cambridge, 2013. xvii+245 pp. ISBN: 978-1-107-02883-8.

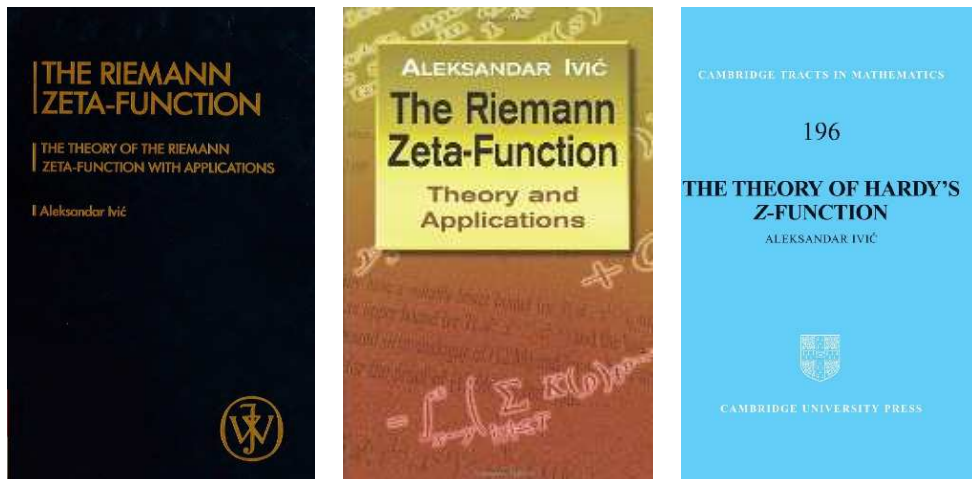


Figure 3: Monographs [3] – The Riemann zeta-function [Editions by John Wiley & Sons (1985) and Dover (2003)] and [5] – The theory of Hardy’s Z -function (Cambridge University Press, 2013)

Monograph [3] related to the famous Riemann zeta-function and the Riemann hypothesis, published by the world-famous publisher John Wiley & Sons, Inc., New York, 1985, is considered as a world key scientific contribution to the field related to Zeta function. This is confirmed by the fact that in 2003, the second edition of the monograph was published, this time by the publishing house “Dover” (see Fig. 3). Moreover, the high citation rate confirmed this. The published books have established a lasting reputation of Academician Ivić as one of the world’s leading scientists in the analytic theory of Riemann’s zeta-function $\zeta(s)$ and related fields of Number Theory. His last book [5] is the unique work written so far that is dedicated to the so-called Hardy’s function $Z(t)$, which plays a key role in studying the zeros of the Riemann zeta-function (see Fig. 3 (right)).

Academician Aleksandar Ivić participated in over 70 scientific conferences, most of which were abroad, mostly as invited lecturer, and in some of them as a plenary lecturer (Quebec(1987), Bordeaux (1996), Kyoto (1996)), Yazd in Iran (2005), Niš (2008), Helsinki (2011), Vrnjačka Banja (2014), Turku (2014), Hsian in China (2014), Gainesville, Florida, USA (2016)).



Figure 4: During a conference at the University of Niš (2008): (Sitting from the left) M. Mateljević, A. Ivić, S. Pilipović, Ć. Dolićanin, G. Milovanović, W. Gautschi; (Standing from the left) M. Ćirić, S. Bogdanović

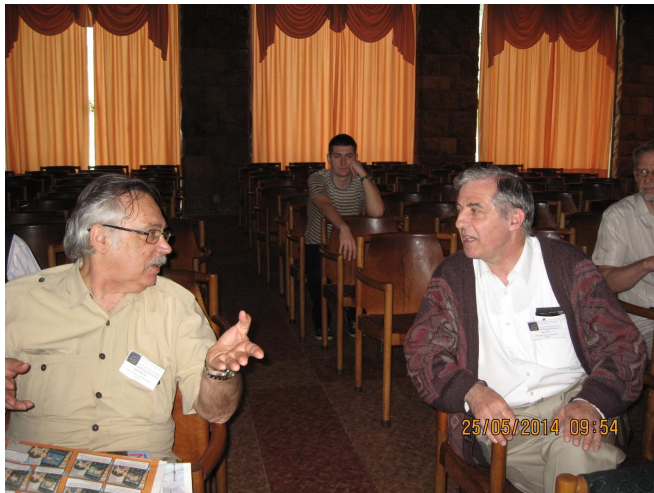


Figure 5: During 13th Serbian Mathematical Congress (Vrnjačka Banja, May 22–25, 2014): Discussion with János Pintz (Hungarian Academy of Sciences)



Figure 6: During a conference at the University of Sad

He participated as many as 13 times in symposia in Oberwolfach, Germany (“Elementary and Analytic Number Theory”, “Exponential Sums”, “The Riemann zeta and allied functions”), Bremen, Bonn, Schloss Schney, Mainz and Hildesheim in Germany, Sofia and Varna (Bulgaria), Geneva (Switzerland), Cardiff (Wales), Cambridge (England), Urbana (USA), Hong Kong, Beijing, Hangzhou (China), Kyoto and Tokyo (Japan), Bordeaux and Luminy (France), Edinburgh (Scotland), Budapest, Lilafired, Keszthely, Noszwa, Debrecen, Visegrád (Hungary), Moscow and St. Petersburg (Russia), Quebec, Vancouver and Ottawa (Canada), Amalfi, Parma, Rome (Italy), Vienna and Strobl (Austria), Seoul (South Korea), Hyderabad (India), Palanga (Lithuania), Madrid (Spain), Turku and Helsinki (Finland), Poznan (Poland), Trondheim (Norway), etc. Some photos are displayed in Figs. 4, 5, 6, 7.

Aleksandar Ivić was a Member of the Mathematical Institute of Serbian Academy of Sciences and Arts and a Member of the editorial board of several international journals: *Publications de l’Institut Mathématique (Beograd)*, *Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat.* [now: *Applicable Analysis and Discrete Mathematics*] and *Proceedings of the Faculty of Science, N. Sad, Ser. Mathematics (Novi Sad)* [now: *Novi Sad Journal of Mathematics*].

He wrote reviews for the world-renowned journals: *Journal of Number Theory*, *Acta Arithmetica*, *Monatshefte für Mathematik*, *Journal für die Reine und Angewandte Mathematik*, *Bulletin of the London Mathematical Society*, etc. As a reviewer for *Mathematical Reviews*, he presented over 120 papers in the period 1978–1993, and wrote more than 550 reviews for *Zentralblatt für Mathematik*.

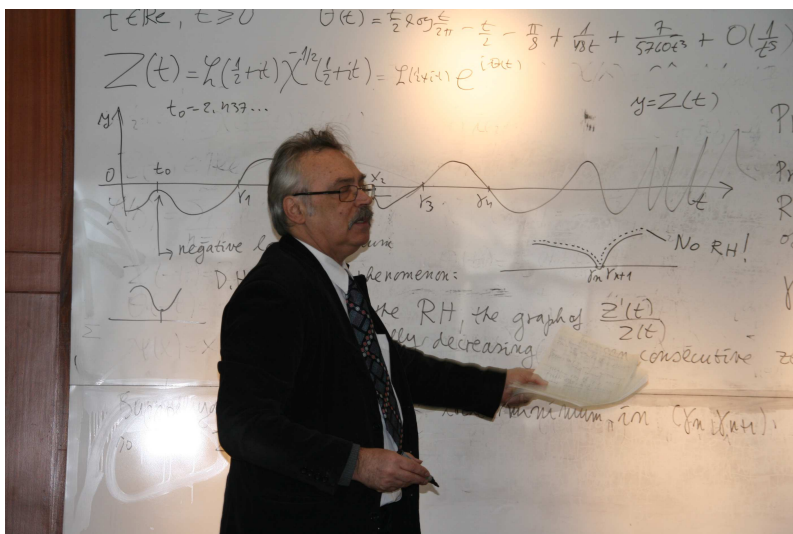


Figure 7: Lecture in Japan



Figure 8: E. Süli, A. Ivić, G.V. Milovanović and D. Cvetković in SASA during a celebration in 2019

Academician Aleksandar Ivić was a co-advisor or a member of the commissions for the defense of six doctoral dissertations and master's theses in Serbia, three master's theses in Hong Kong, and thirteen doctoral dissertations and habilitation theses in France, Finland, Canada, Germany and the Netherlands. Academician Aleksandar Ivić was a visiting professor at the Università d'Genova (1985 and 1989), at the Université de Limoges (1987) and the Tata Institute of Fundamental Research, Mumbai (1990). He was a guest, for two weeks to several months, in well-known mathematical centers and universities in France, Italy, Hungary, USA, Canada, India, Japan, Hong Kong, South Korea, Brazil and elsewhere. He was a guest at the H. Poincaré Institute in Paris (1987, 1990, 1996), IBM Almaden Research Center, San Jose, USA (1989). He spent a month in Rio de Janeiro, Brazil (2015), where he was guest of the Mathematical Institute IMPA - Instituto de Matematica Pura e Aplicada, where he held a mini-course of 20 lectures entitled "Mean values of the Riemann zeta-function".



Figure 9: Sanja with family in the Embassy of Serbia in Rome during the Statehood of Serbia (Sretenje, February 15, 2009)

Academician Aleksandar Ivić was married to Sanda Rašković-Ivić, a prominent psychiatrist, Serbian politician and for a time ambassador to Italy. They have a son Jovan (1997–). From his previous marriage, he has two daughters, Natalija (1980–) and Emilija (1984–), who gave him grandchildren. Natalija has Pavle (11), Marija (8) and Dušan (5), and Emilija has Sofija (11), Marina (7) and Novak (4).



Figure 10: (Left) Wedding of daughter Emilija (December 2009); (Right) Sanja with daughters Emilija and Natalija at the baptism of his grandson Pavle (May 2010)



Figure 11: Sanja with his descendants: son Jovan and daughters Natalija and Emilija, with children (July 2, 2010)



Figure 12: Sanja with his wife Sanda and son Jovan on a trip to Venice (left); Sanja with his wife Sanda in Brazil (right)

Academician Ivić left us too early. We will remember him as a genius mathematician, for his deep and precise scientific results, for his serious scientific work in general, but also as a loyal and reliable friend. Despite his overwhelming scientific esteem, Academician Ivić was a modest and considerate person. He radiated enthusiasm and kind encouragement for the young. His premature departure is a big loss for the Serbian Academy of Sciences and Arts, for Serbian and World science and for all of us. Academician Aleksandar Ivic is noted as one of our greatest mathematicians.

★

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This issue of our journal *Bull. Cl. Sci. Math. Nat. Sci. Math.* is dedicated to the memory of Academician Aleksandar Ivić. It contains eleven papers contributed by colleagues from the Serbian Academy of Sciences and Arts and colleagues who collaborated with Professor Ivić.

Gradimir V. Milovanović
Stevan Pilipović