Mihailo Petrović ALAS

Life Work Times

Serbian Academy of Sciences and Arts

MIHAILO PETROVIĆ ALAS 150th birth anniversary

MIHAILO PETROVIĆ ALAS
150th
birth anniversary

SERBIAN ACADEMY OF SCIENCES AND ARTS

MIHAILO PETROVIĆ ALAS: LIFE, WORK, TIMES

ON THE OCCASION OF THE 150th ANNIVERSARY OF HIS BIRTH

Publisher

Serbian Academy of Sciences and Arts

Knez Mihailova 35, Belgrade

Acting publisher

Academician Vladimir S. Kostić

Editor-in-chief

Academician Marko Anđelković

Editors of publication

Academician Stevan Pilipović

Academician Gradimir V. Milovanović

Professor Dr Žarko Mijajlović

Cover design

Dragana Lacmanović-Lekić

Prepress

Dosije Studio, Belgrade

Selection of artworks

Maja Novaković

English translation

Tatjana Ćosović, Natalija Stepanović

Tanja Ružin Ivanović, Žarko Radovanov, Dora Seleši

Proofreading and editing

Jelena Mitrić

Printing

Planeta print, Belgrade

Print run: 500 copies

ISBN 978-86-7025-818-1

The publication was financially supported by the Ministry of Education, Science and Technological Development of the Republic of Serbia and Telekom Srbija.

MIHAILO PETROVIĆ ALAS
LIFE, WORK, TIMES

ON THE OCCASION OF THE 150th ANNIVERSARY OF HIS BIRTH

SERBIAN ACADEMY OF SCIENCES AND ARTS

Exclusive editions, such as this monograph, call for the engagement, enthusiasm and cooperation of a number of individuals and institutions. We would like to use this opportunity and extend our gratitude to everyone who has taken part or in any way contributed to, or supported the creation and publication of this monograph.

First of all, we would like to express our gratitude to the authors of papers for their effort taken to provide expert and high level insights into some main points of Mihailo Petrović Alas' life and work, at the same time preserving an important aspect of being easy to read and appealing to a broader readership. In addition, we would like to thank to Ms. Snežana Krstić-Bukarica and Ms. Nevena Đurđević from SASA Publishing Section for performing a thorough proofread of the papers, thus making the writing even more articulate.

The monograph features a number of photographs and the copies of documents that have been obtained owing to the kindness of the SASA Archive, SASA Library, SASA Mathematical Institute, Archive of Serbia, Mr. Viktor Lazić from the "Adligat" Society, Mr. Jovan Hans Ivanović and his "Mihailo Petrović Alas" Foundation, "Mihailo Petrović Alas" Primary School, "Svetozar Marković" University Library, Belgrade City Museum, Zavod za udžbenike (Institute for Textbook Publishing) in Belgrade, Virtual Library of Faculty of Mathematics in Belgrade and Digital Legacy of Mihailo Petrović Alas.

The publication of the monograph was financially supported by JP Srbijagas, the Ministry of Education, Science and Technological Development, primarily through scientific projects in which the majority of the authors of the papers takes part, and Telekom Srbija. We would like to express our deep gratitude for their support.

Finally, we would like to express our gratitude to Mr. Mirko Milićević from the publishing house "Dosije Studio" for excellent prepress preparation of the monograph.

S. Pilipović, G. Milovanović, Ž. Mijajlović

CONTENTS

EDITOR'S FOREWORD

As soon as one first encounters the work of Mihailo Petrović, it becomes evident that he was a person that according to its numerous traits was a polymath. Above all, the academician Petrović was a gifted mathematician and a renowned professor at the University of Belgrade, but also a fisherman, writer, philosopher, musician, world traveler and a travel writer. He earned a degree in mathematics at the Belgrade Grand School and a licentiate degree in mathematics, physics and chemistry at the Sorbonne. At the age of 26, only a year after he had completed his studies, he defended his PhD degree in mathematics at the same university, as a student of the famous French mathematicians Henri Poincaré, Charles Hermite and Charles Émile Picard. In the same year (1894) he was elected to the position of professor at the Grand School to which he brought the spirit of the French mathematical school. It was at that point that his long and prolific journey through science began, whereas, owing to him, Belgrade achieved parity with other major European centers in mathematical sciences. He became an initiator and a leader of the Serbian mathematics and strongly contributed to the spirit of the modern European science in Serbia.

Petrović's expertize spanned several mathematical areas in which he achieved scientific results of world-class relevance: differential equations, numerical analysis, theory of functions of a complex variable and geometry of polynomials. He was also interested in natural sciences, chemistry, physics and biology, and he published scientific papers in these fields, too. In his scientific endeavor he managed to meet the most rigorous standards of the most developed European countries. In a brilliant rise, in a few years' time, up to the early 20th century, he wrote around thirty papers that he published in the leading European mathematical journals. It was due to this fact that he was elected a member of the Serbian Royal Academy as early as at the age of 30, and soon after he became a member of a number of foreign academies and prominent expert societies. He won the greatest respect of the global mathematical community: he was among few mathematicians (13) who delivered at least five plenary lectures or lectures as a visiting lecturer at the International Congress of Mathematicians (ICM). He delivered five such lectures (1908, 1912, 1924, 1928 and 1932). One such invitation has been considered by the mathematical community as an equivalent of an induction to a hall of fame. In addition, it has been considered that Petrović was a founder of new scientific disciplines, namely mathematical phenomenology and spectral theory. He invented several analogue computing machines, possessed technical patents and was the main cryptographer of the Serbian and Yugoslav Army.

Up to the Second World War he was the mentor of all doctoral thesis in mathematics defended at the University of Belgrade. Aforementioned is related to one of professor Petrović's greatest and most important achievements – he was a founder of the Serbian mathematical school that has produced a great number of renowned and successful mathematicians not only in Serbia but also around the world.

In 2018, the Serbian Academy of Sciences and Arts and mathematicians in Serbia celebrate the 150th anniversary of the birth of Mihailo Petrović Alas. Throughout this year, the Academy has organized a large exhibition dedicated to Petrović, alongside a solemn gathering and a conference. This monograph commemorates this important jubilee of the Serbian mathematics. Given the fact that a lot of articles on Petrović have already been written, and that his collected works were published at the end of the last century, the editors and authors of the papers in this monograph were faced with a daunting task of finding some new details from professor Petrović's life and career. Even more so given that his body of work is immense, spanning different scientific areas and encompassing topics that at first glance one finds difficult to combine. As Dragan Trifunović, Petrović's biographer and a man who most thoroughly studied his life and work, noted on one occasion that almost an institute was necessary that would encompass professor's entire body of work. Therefore, we set a relatively modest goal to ourselves to shed light upon some main points of Petrović's life and work, times and circumstances he lived in, as well as to elaborate on the present developments in relation to the Serbian mathematical school, through a selection of papers. The authors of the papers steered clear of technical details and excessive use of mathematical language. Hence, the monograph is intended for a broader readership, in particular to those readers who are interested in the history of Serbian science and its evolvement at the turn of the 20th century, but also to those who want to gain a deeper insight into the life of a brilliant mathematician and a polymath, and, we can quite freely say, an unusual personality.

Ž. Mijajlović, S. Pilipović, G. Milovanović

MIHAILO PETROVIĆ ALAS: LIFE AND WORK

MIHAILO PETROVIĆ, ALGEBRAIC GEOMETRY AND DIFFERENTIAL EQUATIONS*

Vladimir DRAGOVIĆ
Mathematical Institute of SASA, Belgrade
The University of Texas at Dallas

This paper is a short excerpt from the lecture the author delivered at the first joint session of all three departments of Mathematical Institute of SASA on 22 May 2018, and within the Round table on the scientific achievements of Mihailo Petrović that was held on the same day in the SASA Gallery. Besides the author of this paper, academicians Stevan Pilipović and Gradimir Milovanović, co-presidents of the ceremony marking the 150th anniversary of the birth of Mika Alas, were also speaking.

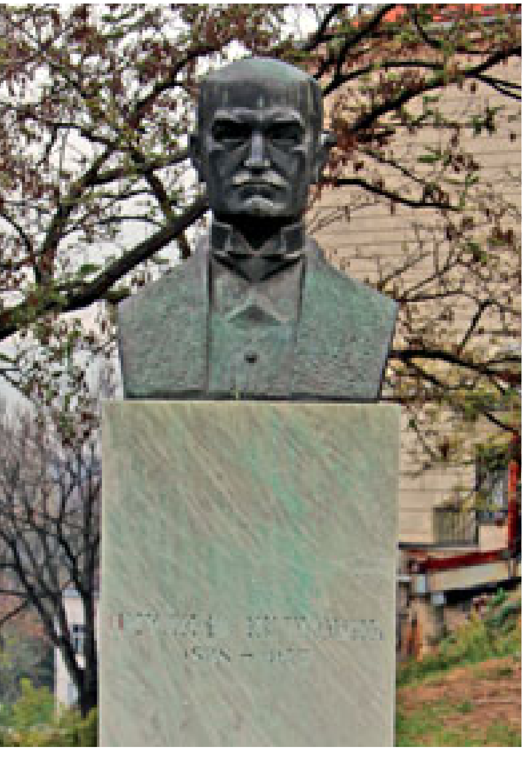
It would be difficult to find another example of a man and a city with an elementary school, a gymnasium and a tavern named after that man. In this city, a real PhD also became a master fisherman. What do we know about the man and his achievements?

We shall elaborate on a series of Mihailo Petrović's achievements incorporating algebraic geometry and analytical theory of differential equations. Those results were achieved within the time frame of almost half a century, from the first half of the 1890s till the end of the 1930s,

* The paper was drafted within the project 174020 "Geometry and Topology of Manifolds, Classical Mechanics and Integrable Dynamical Systems" of the Ministry of Education, Science and Technological Development of the Republic of Serbia. It was supported by the University of Texas at Dallas.

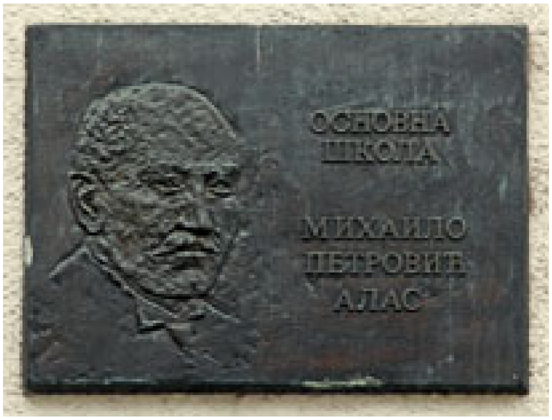
under the strong influence of Petrović's professors Emile Picard and Charles Hermite. Some of them are noted in well-known textbooks and monographs published in France and Russia. Mihailo Petrović passed away in 1943 in Belgrade. In the same city, half a century after Petrović's death, Mathematical Methods of Mechanics Seminar was founded, which primarily focuses on the relation of algebraic geometry and differential equations. Today, twenty five years after the Seminar's foundation, we have just left the door of Petrović's scientific treasury ajar, while wiping the dust off the old manuscripts, thus getting an insight into the work of the great master. We cannot help but wonder how ideas and scientific discoveries have been disseminated and how they travel through space and time.

We live under the well nurtured illusion that the passage of time brings nothing but progress and that scientific progress is a continuous process. Without denying the progress of both science and society, one has to bear in mind that, with the passage of time, many scientific facts, techniques and theories are likely to be forgotten, upstaged, to lose their place in curricula and give way to the latest and more modern scientific discoveries. In science, just like in other areas of creation, there is a trend, which comes and goes, to let, in an inexplicable and unjustified way, valuable achievements fade into oblivion and obscurity. It is our impression that a similar destiny befell the scientific legacy of Mihailo Petrović, regardless of his popularity during his life and after his death, regardless of the number of his students and his students' students, and in spite of his brilliant achievements. We are looking at an impressive list of Petrović's eleven doctoral students and almost nine hundred followers, according to *Mathscinet* data (see detailed overview in professor Pilipović's paper [Pil2018]). This allows the conclusion that the past one century and a half has seen the domination of Mihailo Petrović over Serbian mathematics in the first 75 years, whereas Mika's students and his students' students dominated Serbian mathematics in the remaining 75 years.



A bust of Mihailo Petrović Alas in front of his birthplace, the work of Aleksandar Zarin, 1969

Regardless of this, I can testify that during my student days back in the 1980s, it was only at the lectures within regular course in differential equations held by professor Ljubomir Protić that I could get an insight into the specific scholarly endeavor of Mihailo Petrović and its place in modern science. In September 1988, at Moscow State University, professor Dubrovin asked me to tell him about the significant achievements of Serbian mathematics during my postgraduate interview. With relief, I remembered professor Protić's lectures and talked about the Petrović's method [Pet1896, Pet1899Annalen], which emerged two decades before the similar ideas of great Russian mathematician and mechanic Chaplygin [Chap1919]. (Professor Milorad



Panel at "Mihailo Petrović Alas" Elementary School

Photograph and personal belongings of Mihailo Petrović from "Mihailo Petrović Alas" Elementary School

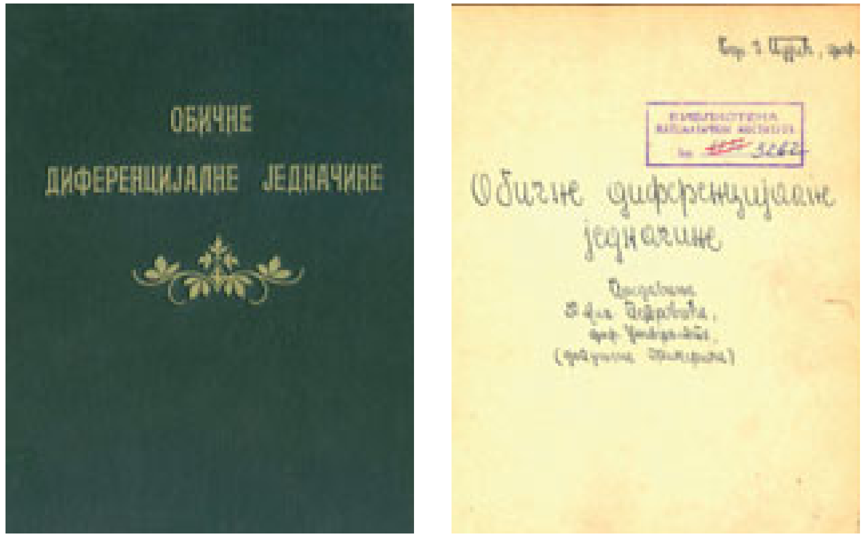
Bertolino had explored Petrović's precedence in this matter for more than 15 years, see [Ber1957, Ber1967, BerTrif1971], [Prot2015], [Pea1886]).

Therefore, it is worth commending the projects that entailed publication of the collected works of Mihailo Petrović in 1999 [Pet1999] and this year's ceremony marking the 150th birth anniversary of our distinguished mathematician, as our collective effort against oblivion and our care for preserving national, but also world scientific legacy.

When preparing myself for my lectures, I avoided consulting *Collected Works* on purpose, prior to choosing the material. For the purpose of this overview, I selected three achievements of Mihailo Petrović in the field of algebraic differential equations. Conditionally, we will name them Theorems A, B and C. As it is well known, Petrović's scope of work and his scientific interests were very broad, nevertheless his theory of algebraic differential equations stands out as his biggest achievement. It turned out that Theorems B and C are from his papers in *Acta mathematica* from 1899 and *Publications* from 1938, which were not included in the *Collected Works* [Petr1999]. Theorem A resulted from a PhD dissertation [Petr1894] that Petrović defended in 1894 at Sorbonne, under the tutelage of Charles Hermite and Emile Picard, with Paul Painleve as the third committee member.

Theorem A deals with the nature of the solution of a generalization of the Riccati differential equation.

An old French proverb says: *anyone can differentiate, but only those who know can integrate*. It was great French mathematician Liouville who, back in the 1830s, brought up the question of the nature of functions that are integrals of some given functions.The idea of studying the fixed and movable singularities of differential equations is dating back to Fuchs. Fixed are those singularities that depend on the differential equation itself and are common for all its solutions,



Cover page of the Book of notes from Petrović's lectures about simple differential equations, around 1910–1914 (Library of MISASA, 3262)

Paul Painlevé (1863–1933), French mathematician

while the movable ones are the ones which change ("move") depending on the initial conditions and which depend on various solutions. The local solutions of differential equations can be analytically extended, but this procedure can lead to multivalued function as a solution of the given equation. Multivaluedness can be tamed by introducing the appropriate Riemann surface as the function domain. If an equation has no *movable* critical singularities in the neighbourhood of which multivaluedness is generated, then such Riemann surface can be chosen as common for all solutions. During the years preceding Petrović's arrival to Paris in 1889, the following fundamental theorems concerning first order differential equations were proved. Henri Poincare and Lazarus Fuchs proved in 1884/85 that among first order equations linear ones stand out, Riccati and Weierstrass functions, as the only ones without movable critical points. Paul Painleve, in his brilliant PhD dissertation in 1887, showed that first order equations cannot have solutions with movable essential singularities. In the year when Petrović arrived in Paris, two very important papers were published [Kow1889, Picard1899]. Both papers are as relevant today as ever and have served as an inspiration to our Seminar and also inspired some of our recent papers. The paper authored by Kowalevski directed analytical theory of differential equations towards applications in mechanics and created foundations of the so-called Kowalevski-Painleve analysis: to describe classes of equations the general solutions of which have certain given analytical characteristics. Kowalevski herself was dealing with the systems of Euler-Poisson equations that describe the dynamics of movement of a heavy firm object around an immovable point under the influence of the force of gravity. She regarded time as a complex variable for the first time in history in a mechanical problem. By examining conditions under which general solutions would have only poles as movable singularities, Kowalevski discovered a new integrable case which is named after her. For this paper, Kowalevski received the Bordin Prize from the French Academy.

In the mentioned Picard's paper, among other things, an algebraic-geometrical solution of the equation, which will be a special case of the future so-called Painleve equations, was found. These second order equations with characteristic that general solutions only have poles as movable singularities were introduced by Painleve at the beginning of the next century [Pain1902], thus marking the beginning of the new era of systematic studying of second order equations with the so-called Painleve characteristic. Painleve's students continued further research concerning second order Painleve eqations, see for instance [Fuchs1906, Gamb1910]. These papers are as relevant today as ever, and have served as a great inspiration for our recent papers.

THEOREM A

Before we formulate the first Petrović's result which we want to show, let us be reminded of the basic results on the Riccati equations. Those are first order equations, where their right side is a quadratic polynomial P per the unknown function w, with coefficients which are meromorphic functions of the independent variable z:

$$w'=P(w,z)$$

If one particular solution of the Riccati equations is known, the equation is reduced to a linear first order equation. Otherwise it is reduced to linear second order equations and has no movable points. If three particular solutions are known w_1, w_2, w_3, then along any of solutions w cross-ratio is constant $w(z):w_1(z):w_2(z):w_3(z)$.

Petrović looked into the generalization of the Riccati equation

$$w' = \frac{P(w,z)}{Q(w,z)}$$

where P, Q are polynomials in w, the coefficients of which are algebraic functions of z.

First he showed that without losing generality, it can be reduced to equations with the form

$$w' = \frac{P_{n+2}(w,z)}{Q_n(w,z)},$$

where P_{n+2}, Q_n are polynomials in w of degree n+2 and n, respectively. Then he decomposed the problem into four sub-cases:

1. case: polynomial equation Q=0 allows more than two different roots $w_i=f_i(z)$;
2. case: polynomial equation Q=0 allows exactly two different roots;

3. case: polynomial equation Q=0 allows exactly one root;
4. case: polynomial Q does not contain w.

Petrović Theorem A (1894): *In case 1 all single-valued solutions are rational. In case 2 all single-valued solutions are reduced to one transcendent function. In case 3 all single-valued solutions are reduced to at most two transcendent functions. Case 4 is in line with the Riccati equation and all solutions are reduced to three transcendent functions.*

In his proof, Petrović skillfully used the known Picard theorems from a complex analysis and the so-called Lemma on critical points:

If the function $f(z,w)$ reaches infinite value at a certain point (z_0,w_0) and if $1/f$ is holomorphic in a neighborhood of the point, the z_0 is a movable critical algebraic point of the equation $w'=f(z,w)$.

Under the condition that P,Q are polynomials in w, the coefficients of which have finitely many isolated singularities, Golubev proved the following theorem:

Golubev Theorem (1911): *If the generalized Riccati equation has three rational solutions, then each single-valued solution is rational.*

Applying the subtle theory of function growth, Malmquist rounded Petrović's research with the following result:

Malmquist Theorem (1914): *If the generalized Riccati equation is not reduced to a Riccati equation, then each single-valued solution is rational function.*

This improves Petrović's achievements in cases 2 and 3. Except in the works of Golubev [Gol1911] and Malmquist [Mal1914], the mentioned Petrović's Theorem A occupies a significant place in the famous monographs authored by Picard and Golubev [Pic1908, Gol1950].

THEOREM B

Before we state the basic result of the only Petrović's paper published in *Acta mathematica*, let us mention two relatively similar results which preceded it, concerning the equations which do not explicitly depend on an independent variable. The aforementioned Weierstrass equation has this form:

$$(z')^2=P_3(z).$$

Its solutions are given in terms of the so-called Weierstrass P-function, which is a double periodic (i.e. elliptic) meromorphic function. The other result comes from one of Petrović's mentors, Hermite:

> **Theorem (Hermite):** *If the solutions of the equation with the form $Q(w,w')=0$, where Q is a polynomial in no w,w' do not have any movable critical points, then they set the curve of genus 0 or 1. The solutions are either rational or depend rationally on exponential or elliptic functions.*

Petrović deals with the problem when equation $Q(w,w', \ldots, w^{(n)})=0$, where Q is a polynomial, has elliptic, i.e. double periodic solutions. He found the necessary geometric conditions, using the polygon technology that he developed in his PhD dissertation [Petr1894]. Based on a polynomial Q polygon Π is constructed. The vertices of the polygon can be simple or multiple. To each multiple vertex D a characteristic polynomial k_D is assigned. The polynomial Q can be presented in the form of the sum s of addends with the form of

$$S_i\, y^{m_{0i}} y'^{m_{1i}} \ldots y^{(p)m_{pi}},$$

where S_i are constants.

The vertices of the polygon are determined by formulas $(M_i,\ N_i)\, i=1,\ \ldots,\ s$.

$$M_i=m_{0i}+m_{1i}+\cdots+m_{pi};$$

$$N_i=m_{1i}+2m_{2i}+\cdots+pm_{pi}.$$

It may happen that for various $i,\ j$ pairs coincide $(M_i,N_i)=(M_j,N_j)$.

Then the corresponding vertex is *multiple*. Petrović was developing theory of such polygons, which generalize the Newton polygons and are adjusted to algebraic differential equations, and made substantial contributions to the theory in his dissertation (see also [Stan1999]). "Characteristic B" which exists in the theorem, also stems from the mentioned thesis.

> **Theorem B (Petrović 1899):** *If the equation $Q=0$ has an elliptic solution, then the polygon has "characteristic B": either it has an edge with a negative integer angle coefficient or it has a multiple vertex D, so that the characteristic polynomial of vertex k_D has at least one whole-number root that is located between the angle coefficients of the edges that meet in the vertex D.*

Example: let us apply the Theorem to the equation $P_m(y'')=Q_n(y)$, where $P_m,\ Q_n$ are given polynomials of a variable of degree m,n respectively. Polygon Π is a triangle $\triangle ABC$ the vertices of which are $A(0,0),\ B(n,0),\ C(m,\ 2m)$.

Condition B here implies that the triangle △ABC has to be an acute triangle. The only edge with negative coefficient is BC, if $n>m$. Angle coefficient is $2m/(m-n)\in Z$. So, $n>m$, $2m/(m-n)\in Z$. Examples of the pairs are: $(m, n) \in \{(1, 2), (1, 3), (2, 4), (2, 6)$.

THEOREM C

In his paper [Pet1938], Petrović showed this very general and elegant theorem:

> **Theorem C** [Petrović 1938] *There is a series of functions $u_1(t)$, $u_2(t)$,... so that for each algebraic differential equation by $x,\ y,\ y',\dots y^{(p)}$ an independent variable x and a solution y can be expressed as the function of parameter t via the finite number of functions $u_i(t)$ in quadrature or without quadrature.*

Functions are determined as the solutions of series of systems, for each natural number p there are $2^{(p^2)}$ systems

$$\frac{du_i}{dt} = \sum_{j=1}^{p} f_i^j e^{u_j}, i, j, = 1, \dots p, \text{ where } f_i^j \in \{0,1\}.$$

At the very end of his paper, Petrović noticed that theorems of similar type concerning the reduction of algebraic differential equations were proven by Appelrot and Lagutinski. Since he had no opportunity to see those papers and did not know their claims or proofs, he said that he could not use or quote them.

We were reading and applying Appelrot's papers with pleasure. Petrović's remark on Lagutinski reminded me of one nice encounter in Paris ten years ago, with French mathematician of Polish origin, Jean-Marie Strelcin. On that occasion, Professor Strelcin gave me the collected works of Lagutinski. With great enthusiasm he pointed at the importance of the almost forgotten brilliant papers of the talented Russian mathematician who died too early. Intrigued by Petrović's remark, I browsed Lagutisnki's papers and found references which Petrović was missing: [Lag1911, App1924].

Finally, let us remind that Petrović enjoyed a well-deserved international reputation, which is confirmed by the fact that he had been invited to deliver lectures at the International Congresses of Mathematicians five times: Rome 1908, Cambridge 1912, Toronto 1924, Bologna 1928 and Zurich 1932. Nowadays, probably no living mathematician was given such honor. It happened in the early 20th century, but very seldom.

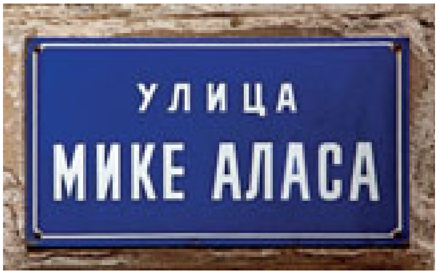
OUR DAYS

The monograph titled *First Integrals with Restrictions* [Petr1999, vol. 2, 159–202] (*Integrales premieres a restriciones*) from 1929 is very similar by its content to the theory of integrable dynamic systems, which is the main topic of our Seminar. On pages 162–163 Petrović mentioned the canonical equations of dynamics (Hamiltonian systems) and formulated the fundamental Liouville theorem. Petrović's integrals with restrictions are very similar to invariant relations that were, among others, researched by Appelrot, and also by us. Our road to the theory of integrable systems would have been much easier if we had read this Petrović's paper in our young days.

Post festum. Further to my question from the lecture to find out more about Mika's older Parisian colleague Mijalko Ćirić, who is mentioned at *Mathscinet* as Charles Hermite's doctoral student, my colleague, Miloš Milovanović, PhD, responded. He kindly provided a link to Radoje Domanović's story *He Abolished Mechanics* from *Stradija* dated March 20, 1905:

https://domanovic.wordpress.com/tag/mijalko-ciric/

The story is very interesting for its juicy, sharp and as relevant as ever Domanović's language, but also because of its moral that being a student of a great mathematician and being a "Parisian" in Belgrade at the beginning of the 20th century did not automatically guarantee a successful academic career. This does not represent the views of the author of this article or of the Editorial board on historical persons that are mentioned in Domanović's text and on their creative achievements and merits.



Mika Alas Street in Dorćol

REFERENCES

[App1924] Аппелрот, Г., *Мат. сборник*, т. 32, 1924, 9–21.

[Ber1957] Бертолино, М., Неке функционалне неједнакости добијене применом Чаплигинове методе и упоређивање са резултатима М. Петровића, *Весник Друштва МФ*, Београд 1957, 87–94.

[Ber1967] Bertolino, M., Priorite de Michel Petrovitch relative an theoreme de Tchaplyguine sur les integralites differentielles du premier ordere, *Mat. vesnik*, 4, 1967, 165–168.

[BerTrif1971] Bertolino, M., Trifunović D., Sur le theoreme fondamentale de S. A. Čapligin sur l'integralite differentielle du premier ordre, *Mathematica balcanica*, 1971, 11–18.

[Chap1919] Чаплыгин, С. А., *Основания новоīо сӣособа ӣриближенноīо инӣеīрирования дифференциальных уравнений*, 1919.

[Fuch1906] Richard, F., *Sur quelques equations differentielles lineaires du second ordre, Comptes Rendus*, 1906.

[Gamb1910] Gambier, B., Sur les équations différentielles du troisième ordre et d'ordre supérieur dont l'intégrale générale a ses points critiques fixes, *Acta Mathematica*, 1910.

[Gol1950] Голубев, В. В., *Лекции ӣо аналиӣической ӣеории дифференциальных уравнений*, Москва 1950.

[Gol1911] Голубев, В. В., Об одном приложении теоремы Picard'а к теории дифф. уравнений, *Маӣ сборник*, 1911.

[Kow1899] Kowalevski, S., Sur le probleme de la rotation d'un corps solide autour d'un point fixe, *Acta Mathematica*, 1899.

[Lag1911] Лагутинский, Н., *Маӣ. сборник*, т. 27, 1909–1911, 420–423.

[Mal1914] Malmquist, I., Sur les fonctions a un nombre fini de branches definies par les equations differentielles du premier ordre, *Acta Mathematica*, V. 36, 1914.

[Pain1902] Painleve, P., Sur les equations differentielles du second ordre et d'ordre superieur dont l'integrale generale est uniforme, *Acta Mathematica*, 1902.

[Pea1886] Peano, G., *Sull' interabilita delle equazioni differenziali di primo ordine*, Torino, 1886.

[Petr1894] Petrovitch, M., *Sur les zeros et les infinis des integrales des equations differentielles algebriques*, Sorbonne, 1894.

[Petr1899Annalen] Petrovitch M., Sur une maniere d'etendre le theorem de la moyenne aux equations differentielles du premier ordre, *Math. Annalen*, 1899.

[Petr1899] Petrovitch, M., Sur une propriete des equations diff. a l'aide des fonctions meromorphs doublement periodiques, *Acta Mathematica*, 1899.

[Petr2018] Petrovitch, M,. On a property of differential equations integrable using meromorphic double-periodic functions, *Theoretical and Applied Mechanics*, 2018, vol. 45 (2018) Issue 1, 121–127, DOI: https://doi.org/10.2298/TAM1801121P.

[Petr1929] Petrovitch, M., *Integrales premieres a restriciones*, 1929.

[Petr1938] Petrovitch, M., Theoremes generaux sur les equations differentielles algebriques, *Publications de l'Institut Mathématique*, 1938, t. 6–7, 290–325.

[Petr1999] *Сабрана дела Михаила Пеӣровића*: у 15 томова, Завод за издавање уџбеника, Београд, 1999.

[Pic1899] Picard, E., Memoire sur la theorie des fonctions algebriques de deux variables, J. de *Math. pures appl*., 1899.

[Pic1908] E. Picard, *Traite d'analyse*, Paris 1908, III, 378.

[Pil2018] Пилиповић, С., *Академик Михаило Пеӣровић – доӣриноси у науци и насӣави*, 2018.

[Prot2015] Протић, Љ., Живот, дело и научни рад Михаила Петровића Аласа у: *Срӣски маӣемаӣичари*, зборник предавања одржаних на скупу Српски математичари у оквиру манифестације Мај месец математике 2012, САНУ, Универзитет у Београду, Завод за уџбенике, Београд 2015, 17–28.

[Stan1999] Станковић, Б., *Аналиӣичка ӣеорија диференцијалних једначина Михаила Пеӣровића*, Сабрана дела Михаила Петровића, том 1, 367–378.