



Mihailo Petrović

ALAS

Life
Work
Times



Serbian Academy of Sciences and Arts







SERBIAN ACADEMY OF SCIENCES AND ARTS

MIHAILO PETROVIĆ ALAS: LIFE, WORK, TIMES
ON THE OCCASION OF THE 150th ANNIVERSARY OF HIS BIRTH

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LIFE, WORK, TIMES

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OF HIS BIRTH



SERBIAN ACADEMY OF SCIENCES AND ARTS

Exclusive editions, such as this monograph, call for the engagement, enthusiasm and cooperation of a number of individuals and institutions. We would like to use this opportunity and extend our gratitude to everyone who has taken part or in any way contributed to, or supported the creation and publication of this monograph.

First of all, we would like to express our gratitude to the authors of papers for their effort taken to provide expert and high level insights into some main points of Mihailo Petrović Alas' life and work, at the same time preserving an important aspect of being easy to read and appealing to a broader readership. In addition, we would like to thank to Ms. Snežana Krstić-Bukarica and Ms. Nevena Đurđević from SASA Publishing Section for performing a thorough proofread of the papers, thus making the writing even more articulate.

The monograph features a number of photographs and the copies of documents that have been obtained owing to the kindness of the SASA Archive, SASA Library, SASA Mathematical Institute, Archive of Serbia, Mr. Viktor Lazić from the "Adligat" Society, Mr. Jovan Hans Ivanović and his "Mihailo Petrović Alas" Foundation, "Mihailo Petrović Alas" Primary School, "Svetozar Marković" University Library, Belgrade City Museum, Zavod za udžbenike (Institute for Textbook Publishing) in Belgrade, Virtual Library of Faculty of Mathematics in Belgrade and Digital Legacy of Mihailo Petrović Alas.

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S. Pilipović, G. Milovanović, Ž. Mijajlović

CONTENTS

7 | Editor's foreword

MIHAILO PETROVIĆ ALAS: LIFE AND WORK

- 13 | Žarko Mijajlović, *Mihailo Petrović Alas and His Age*
35 | Stevan Pilipović, *Academician Mihailo Petrović – His Contributions to Science and Education*
65 | Gradimir V. Milovanović, Miodrag Mateljević, Miloljub Albijanić, *The Serbian School of Mathematics – from Mihailo Petrović to the Shanghai List*
93 | Vojislav Andrić, *Pedagogical Work of Mihailo Petrović*

MIHAILO PETROVIĆ IN PHILOSOPHY, LITERATURE AND PUBLIC LIFE

- 115 | Slobodan Vujošević, *Mathematical Phenomenology and the Philosophy of Mathematics*
127 | Nikola Petrović Morena, *Mathematical Phenomenology between Myth and Reality*
143 | Đorđe Vidanović, *Mihailo Petrović Alas and Modern Cognitive Science*
157 | Mihajlo Pantić, *On Fishing and Literary Works of Mihailo Petrović Alas*
171 | Milan Božić, *Travels and Travelogues*
185 | Nenad Teofanov, *Mihailo Petrović's Fishing – One View*

MIHAILO PETROVIĆ: INVENTIONS AND PATENTS

- 201 | Radomir S. Stanković, *The Hydrintegrator of Mihailo Petrović Alas*
215 | Katica R. (Stevanović) Hedrih, *Mechanics and Engineering in Mihailo Petrović's Work*
233 | Miodrag J. Mihaljević, *Mihailo Petrović Alas and the State Cryptography of the Interwar Period*

MATHEMATICAL LEGACY OF MIHAILO PETROVIĆ, APPENDICES

- 249 | Zoran Ognjanović, *Tadija Pejović and the Logical Branch of Mihailo Petrović Alas' Successors*
257 | Vladimir Dragović, *Mihailo Petrović, Algebraic Geometry and Differential Equations*

- 267 | Nataša Krejić, *Group for Numerical Mathematics in Novi Sad*
275 | Dora Seleši, *Mihailo Petrović Alas – Scientific Legacy and Modern Achievements in Probability Theory*

MIHAILO PETROVIĆ IN THE MEDIA AND ARCHIVES

- 285 | Maja Novaković, *Digitization of the Legacy of Mihailo Petrović Alas*
299 | Marija Šegan-Radonjić, *Documents on Mihailo Petrović Alas in the Archives of the Mathematical Institute SASA (1946–1954)*

GENEALOGY

- 309 | Boško Jovanović, *Mathematical Genealogy of Mihailo Petrović Alas*
329 | *Mathematical Genealogical Tree of Mihailo Petrović*, compiled by Žarko Mijajlović
347 | Remarks

MIHAILO PETROVIĆ: SELECTED BIBLIOGRAPHY

- 359 | *Appendices to Bibliography and Sources of Data*, prepared by Žarko Mijajlović and Stevan Pilipović

EDITOR'S FOREWORD

As soon as one first encounters the work of Mihailo Petrović, it becomes evident that he was a person that according to its numerous traits was a polymath. Above all, the academician Petrović was a gifted mathematician and a renowned professor at the University of Belgrade, but also a fisherman, writer, philosopher, musician, world traveler and a travel writer. He earned a degree in mathematics at the Belgrade Grand School and a licentiate degree in mathematics, physics and chemistry at the Sorbonne. At the age of 26, only a year after he had completed his studies, he defended his PhD degree in mathematics at the same university, as a student of the famous French mathematicians Henri Poincaré, Charles Hermite and Charles Émile Picard. In the same year (1894) he was elected to the position of professor at the Grand School to which he brought the spirit of the French mathematical school. It was at that point that his long and prolific journey through science began, whereas, owing to him, Belgrade achieved parity with other major European centers in mathematical sciences. He became an initiator and a leader of the Serbian mathematics and strongly contributed to the spirit of the modern European science in Serbia.

Petrović's expertise spanned several mathematical areas in which he achieved scientific results of world-class relevance: differential equations, numerical analysis, theory of functions of a complex variable and geometry of polynomials. He was also interested in natural sciences, chemistry, physics and biology, and he published scientific papers in these fields, too. In his scientific endeavor he managed to meet the most rigorous standards of the most developed European countries. In a brilliant rise, in a few years' time, up to the early 20th century, he wrote around thirty papers that he published in the leading European mathematical journals. It was due to this fact that he was elected a member of the Serbian Royal Academy as early as at the age of 30, and soon after he became a member of a number of foreign academies and prominent expert societies. He won the greatest respect of the global mathematical community: he was among few mathematicians (13) who delivered at least five plenary lectures or lectures as a visiting lecturer at the International Congress of Mathematicians (ICM). He delivered five such lectures (1908, 1912, 1924, 1928 and 1932). One such invitation has been considered by the mathematical community as an equivalent of an induction to a hall of fame. In addition, it has been considered that Petrović was a founder of new scientific disciplines, namely mathematical phenomenology and spectral theory. He invented several analogue computing machines, possessed technical patents and was the main cryptographer of the Serbian and Yugoslav Army.

Up to the Second World War he was the mentor of all doctoral thesis in mathematics defended at the University of Belgrade. Aforementioned is related to one of professor Petrović's greatest and most important achievements – he was a founder of the Serbian mathematical school that has produced a great number of renowned and successful mathematicians not only in Serbia but also around the world.

In 2018, the Serbian Academy of Sciences and Arts and mathematicians in Serbia celebrate the 150th anniversary of the birth of Mihailo Petrović Alas. Throughout this year, the Academy has organized a large exhibition dedicated to Petrović, alongside a solemn gathering and a conference. This monograph commemorates this important jubilee of the Serbian mathematics. Given the fact that a lot of articles on Petrović have already been written, and that his collected works were published at the end of the last century, the editors and authors of the papers in this monograph were faced with a daunting task of finding some new details from professor Petrović's life and career. Even more so given that his body of work is immense, spanning different scientific areas and encompassing topics that at first glance one finds difficult to combine. As Dragan Trifunović, Petrović's biographer and a man who most thoroughly studied his life and work, noted on one occasion that almost an institute was necessary that would encompass professor's entire body of work. Therefore, we set a relatively modest goal to ourselves to shed light upon some main points of Petrović's life and work, times and circumstances he lived in, as well as to elaborate on the present developments in relation to the Serbian mathematical school, through a selection of papers. The authors of the papers steered clear of technical details and excessive use of mathematical language. Hence, the monograph is intended for a broader readership, in particular to those readers who are interested in the history of Serbian science and its evolvement at the turn of the 20th century, but also to those who want to gain a deeper insight into the life of a brilliant mathematician and a polymath, and, we can quite freely say, an unusual personality.

Ž. Mijajlović, S. Pilipović, G. Milovanović



MIHAILO PETROVIĆ ALAS:
LIFE AND WORK

PEDAGOGICAL WORK OF MIHAILO PETROVIĆ*

Vojislav ANDRIĆ
Society of Mathematicians of Serbia
Valjevo Gymnasium

“Serene, silent, modest, humanly simple, superhumanly endowed, Mihailo Petrović was one of the greatest sons of our nation.”

Milutin Milanković

Various ranking lists with one hundred best, greatest, most important or most influential persons are often prepared across the world. Of course, such choices are invariably somewhat disputable and partially biased, depending on the authors making such lists. One thing, however, is certain – the list of the leading and most important Serbian scientists, greatest mathematicians, the most versatile and unusual people, and the list of one hundred most important historical figures among the Serbs of all time will certainly contain the name of Mihailo Petrović Alas – a man who significantly marked the time in which he lived and enriched the culture of our people with his prolific work in a large number of disciplines.

Even a fragmentary study of the work of Mihailo Petrović makes every researcher convinced that he was an outstanding person, teacher and scientist. Just as his entire life and work were marked by incredible and unusual versatility, his pedagogical work was also characterised by

* I dedicate this work to late professor dr Dragan Trifunović, who did the most to shed light on the interesting and dynamic life of Mihailo Petrović, his versatile personality and enormous contributions to the history and culture of our people.



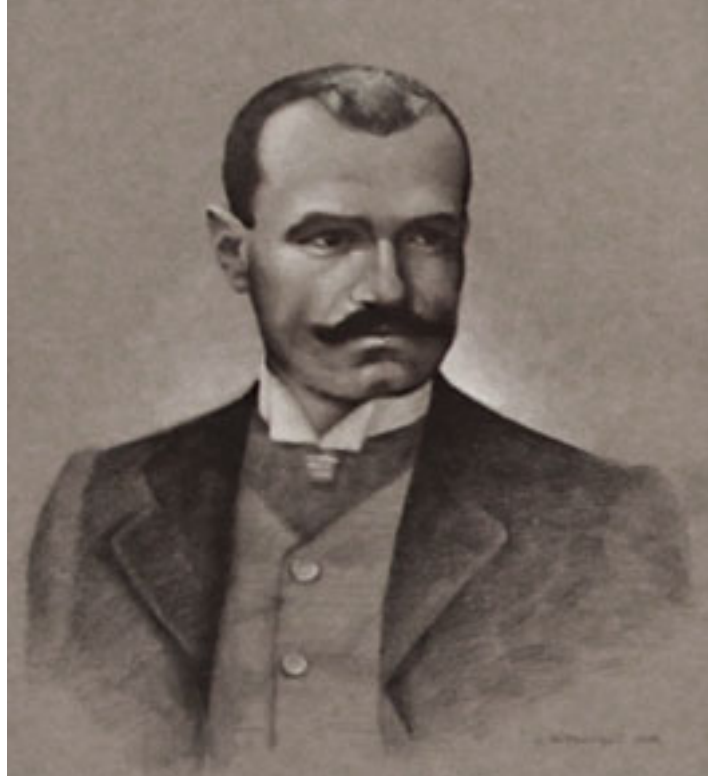


House of Mihailo Petrović Alas
at 22, Kosančićev venac st.

many significant contributions to the future of our people and civilisation in general.

Of around twenty areas of mathematics and numerous disciplines outside of mathematics that Mihailo Petrović Alas successfully dealt with, mathematical science and teaching of mathematics are certainly among the most important, all the more so as the majority of scientists – teachers find it hard to say where teaching stops and science starts, because successful professional and teaching activity implies primarily a high degree of scientific expertise and a solid mathematical apparatus, including didactical skills of the lecturer.

The aim of this article is to elucidate the pedagogical work of Mihailo Petrović Alas and his indirect and direct contribution to the development of the teaching of mathematics in Serbia and the world, in his time and later, when the implications of his work became fully manifest in Serbia and Yugoslavia.



Mihailo Petrović's portrait, a drawing from "Mihailo Petrović Alas" Primary School

TEACHING

Mihailo Petrović's pedagogical work deserves to be elucidated from all aspects, most notably his teaching efforts, the courses he taught, his direct influence on the creation of the Belgrade Mathematical School, contributions to the progress of several generations of his students and doctors of science in the fields of teaching and science, and the promotion of mathematics in Serbia and the region.

Very young, aged only 26, Mihailo Petrović became a full professor at the Faculty of Philosophy of the Belgrade Great School.⁴⁹ In the academic 1894/95 year, he taught mathematical analysis at the Natural-Sciences Department of the Faculty of Philosophy. The documents available to us show that professors at higher educational institutions were at the time obliged to take an oath. Newly elected professors would also be received in audience with the ruler.⁴⁹

In the June term of 1895, Mihailo Petrović had an exam in mathematical analysis and in the academic 1895/96 year, he taught mathematics II and III at the Faculty of Philosophy and Technical Sciences of the Great School. In this period, Mihailo Petrović often presided at exams of his other colleagues, particularly Bogdan Gavrilović, and at exams in physics, including even the French language. From 1905 to 1909 (when Milutin Milanković was engaged to teach at the Applied Mathematics Department)⁴⁹, Mihailo Petrović was the only professor of mathematics at the Faculty of Philosophy and, together with Bogdan Gavrilović, the only mathematician at

the Great School. At the time, following the model of French university studies of mathematics, Petrović began to teach his specialised mathematical courses.

It is worth noting that the rule on the limited number of teachers⁴⁹ was in force at the Great School at the time. It was only in 1912 that professor Mladen Berić was received as the first associate to Mihailo Petrović. Later, after World War I, the Mathematics Department was systematically reinforced. Teaching professors were Anton Bilimović, Nikola Saltikov, Radivoj Kašanin, Jovan Karamata and others.

Mihailo Petrović loved his teaching profession and dedicated to it a lot of time during his professional career (formally 44 years: from 1894 to 1938). His lectures were simple and appealing to students. The following quotation speaks volumes about him as a lecturer and pedagogue, who always held a piece of chalk: “His school produced forty classes of mathematicians, all of whom cherished and still cherish undivided respect and esteem towards their professor, who instilled them with knowledge at his versatile courses in mathematical analysis, which were presented with ease and reflected his deep thought and virtuosity of teaching. He gladly received, invited and taught everyone who showed interest in mathematics. He wanted to unselfishly devote himself to teaching and science in our country.”⁵⁰

At the start of his professorship, Mihailo Petrović was considered someone with a very strict grading criterion. The highest mark (five) was very rare, with a half of students often referred to the following teaching terms. Such criterion considerably differed from the hitherto practice and significantly encouraged students of mathematics and technical sciences to prepare themselves more systematically for the exams in mathematics and mathematical sciences.⁵¹ Mihailo Petrović remained strict primarily towards himself and then towards others until the end of his life.

Petrović was a sincere and eager devotee to science in general, particularly mathematical and natural sciences. His biographers mention his great interest in all areas of mathematics and some restraint in terms of the contents of geometries and probability theories. In the course of his 44-year long career of a full professor at the Great School and University of Belgrade (from 1894 to 1938) and an honorary professor (from his retirement until April 1941), Mihailo Petrović taught a number of regular and specialised courses: analytic geometry in plane and space, higher algebra, differential and integral calculus, geometrical application of the theory of differential equations, calculation with numerical intervals, infinite order theory, elliptic functions, partial differential equations in mathematics and physics, second-order linear differential equation and its application, qualitative integration of differential equations, integration of differential equations by use of series, analytical problems for interpretation, theory of errors, theory of analytical functions, elements of mathematical phenomenology⁵².

The range of areas and disciplines taught by Mihailo Petrović best testifies to his mathematical knowledge, and creative and teaching originality as many of the above courses were not classical, but were based on Petrović's long-lasting teaching and research work and innovations



Jovan Karamata (1902–1967)



Tadija Pejović (1892–1982)



Dragoslav Mitrinović (1908–1995)

that he continuously introduced in his lectures, based on his papers and papers published in foreign journals. Petrović's courses were simple, short, concise and methodologically well-designed, without any exaggeration in content.⁵³ Lecture notes followed each of the above courses, to assist students in learning.⁵⁴

Mihailo Petrović carefully followed, detected and supported his best students. After completing the studies of mathematics at the Belgrade Faculty of Philosophy, his students would most often be referred to Belgrade gymnasiums as teachers of mathematics, while the best ones were engaged as assistants at the University. In the 1912–1938 period, eleven mathematicians defended their doctoral theses before Mihailo Petrović. In the decades before and after World War II, these mathematicians, together with their students and associates, played a great role in the development of mathematical science and the teaching of mathematics in Serbia and Yugoslavia.

Mihailo Petrović's doctoral students were the following (by order of gaining the doctoral title): Mladen Berić (1912), Sima Marković (1913), Tadija Pejović (1923), Radivoj Kašanin (1924), Jovan Karamata (1926), Miloš Radojičić (1928), Dragoslav Mitrinović (1933), Konstantin Orlov (1934), Danilo Mihnjević (1934), Petar Muzen (1937), Dragoljub Marković (1938).

The analysis of the available databases shows that as much as it is important to observe the doctoral students of Mihailo Petrović, so much it is important to analyse their students who continued their professors' curricular, extra-curricular and scientific activities. Mihailo Petrović currently has close to nine hundred mathematical "successors" given that his doctoral students, following in the footsteps of their professor, aimed to leave behind themselves numerous successors of their work and ideas. The most prolific in "producing" the scientific offspring were Jovan Karamata (12 doctoral students and 480 successors), Tadija Pejović (17 doctoral students and 193 successors) and Dragoslav Mitrinović (33 doctoral students and 120 successors).⁵⁵



Bogdan Gavrilović (1864–1947)

Although some connoisseurs of Mihailo Petrović's life and work are sceptical in terms of the scientific formation and continuous instruction of younger colleagues, there is no disputing that Mihailo Petrović, his closest associates Bogdan Gavrilović and Milutin Milanković, and his doctoral students and closest associates after the Great War, contributed the most to the creation and development of the institution designated in the history of our science as the Belgrade Mathematical School.

It is considered that the formal beginnings of the Belgrade Mathematical School date to the time when the first doctoral student of Mihailo Petrović – Mladen Berić began to work at the University of Belgrade. Berić defended his doctoral dissertation in May 1912 before a committee consisting of Mihailo Petrović and Milutin Milanković.⁵⁶

However, equally important for the functioning and activity of the Belgrade Mathematical School was the arrival of professor Milutin Milanković to Belgrade University (the Applied Mathematics Department – 1909), Anton Bilimović (1920) and Nikolaj Saltikov (1921). In the third decade of last century, a team of highly capable mathematicians and applied mathematicians gathered around Mihailo Petrović. They initiated many useful mathematical activities such as: replenishing the university mathematical library, set up by Mihailo Petrović, writing quality secondary-school textbooks, launching the *Mathematical Journal* etc., which will be elaborated in more detail hereinafter.⁵⁶

Furthermore, the Belgrade Mathematical School was the precursor of the membership of our mathematicians in international mathematical associations. In late 1921, upon the order of the Serbian Royal Academy, Mihailo Petrović and Bogdan Gavrilović wrote a report, proposing and recommending that the National Section of the International Mathematical Union (IMU) be established within the Professorial Society.⁵⁷

One of the most important contributions of the Belgrade Mathematical School was certainly the initiation of the mathematical journal in foreign languages – *Publication mathematiques de l'Université de Belgrade*. Upon the initiative of Mihailo Petrović and Milutin Milanković, the journal was established in 1932 to help interested mathematicians actively involved in research in the fields of mathematics and applied mathematics to publish their results. Until 1941, seven issues were published. The journal was, in fact, the gazette of the Belgrade Mathematical School. Owing to it, our mathematicians could present themselves, individually and collectively, to the international mathematical public⁵⁸.

The journal continued to be published in 1947 under the title: *Publications de l'Institut Mathématique (nouvelle serie)*. It is issued twice a year by the Mathematical Institute of the Serbian Academy of Sciences and Arts in Belgrade⁵⁹. The new journal series has been published since 1961. The journal accepts papers of Serbian and foreign mathematicians. The main language is English. Papers in French, Russian and German are also accepted. The journal is one of the leading Serbian scientific journals in the field of mathematics and enjoys considerable reputation in the country and the world. It has a wide range of associates and a respectable editorial board.

“Mihailo Petrović rejoiced in his students’ scientific success and did not impose upon them fields of research.”⁶⁰ He was very circumspect and modest, never aiming to ascribe to himself the success and results of his associates, while at the same time not restraining himself from putting everyone in their place. Quite illustrative is his answer to journalist Krsto Cicvarić, published in the *Politika* daily on 24 June 1921 – Alas wrote that Cicvarić was not competent enough to assess (or even praise) him and his colleagues, some of whom did a lot for science and whom he (Alas) particularly esteemed and loved.⁶¹



Milutin Milanković (1879–1958)



Cover pages of other publications of Mihailo Petrović's textbooks

TEXTBOOKS AND LECTURE NOTES OF MIHAILO PETROVIĆ

It is known, and has already been highlighted in this article, that materials supporting the mathematical courses taught by Mihailo Petrović were mainly sheets (lecture notes). Unlike his colleague Bogdan Gavrilović, he long resisted the writing of a textbook.

In the period from 1909 until the end of Petrović's professorial career, his courses were supported by lithographic sheets (lecture notes). The notes were designed in the following way: students would take detailed notes at Petrović's courses and submit them to Petrović for analysis. He systematically inspected the notes and approved them for lithography (copying). The year 1924 was important as the notes for six Petrović's courses were lithographed. From 1925 to 1930, the notes for all 15 mathematical courses taught by Mihailo Petrović were issued. Unlike the previous ones, these notes were authorised and, in technical terms, the format of hand-written sheets was abandoned, with the text now taking the shape of typewritten books.⁶²

The notes were divided into chapters. Within each chapter, Mihailo Petrović presented theory for each thematic unit (definitions, theorems with evidence), which was followed by solved examples and a significant number of tasks methodologically ranked from simpler to more difficult. In regard to theory, Petrović referred students in his notes to literature with detailed evidence, and in regard to tasks he referred them to more in-depth collections of tasks.⁶¹

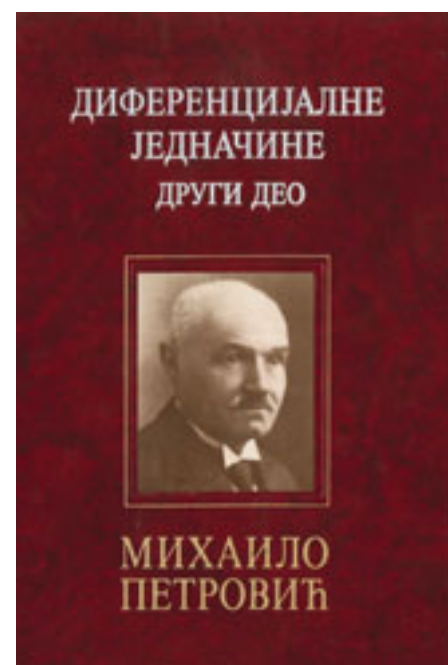
By the end of his career and life, Mihailo Petrović “yielded” and wrote several textbooks: *Calculation with Numerical Intervals* (1932), *Elliptic Functions* (1937) and *Integration of Differential Equations by Use of Series* (1938). Just like some of connoisseurs of his works, he considered his seminal work *Elements of Mathematical Phenomenology* from 1911 to be a textbook.⁶¹

In April 1932, Mihailo Petrović’s textbook *Calculation with Numerical Intervals* was published, containing 193 + II pages, in 15.9 × 23.7 format.⁶¹ The textbook was published by the Endowment of Luka Ćelović Trebinjac within the series “Lectures at Belgrade University”, most probably in 500 copies.⁶¹ Some mathematicians believe that this textbook has all features of a monograph, though this cannot be reliably claimed given the absence of the bibliography of works.

In 1937, Petrović’s textbook *Elliptic Functions* was also published within the series “Lectures at Belgrade University”, by the Endowment of Luka Ćelović Trebinjac, in 128 + III pages, 15.9 × 23.7 format.⁶³ It is interesting that students, not knowing that professor Petrović was working on the textbook, prepared and published the new issue of lecture notes on the same topic. It is noteworthy that a significant role in preparing the textbook was played by Petrović’s doctoral students and associates Dragoslav S. Mitrinović, who carried out detailed proofreading, and Miloš Radojčić, who made the final review, together with his professor.

Already the following, 1938 year, the textbook *Integration of Differential Equations by Use of Series* was also printed, again by the Endowment of Luka Ćelović Trebinjac, within the same series⁶⁴. It had 219 pages, 15.2 × 23.3 format.

These three textbooks of Mihailo Petrović were published three times (“Naučna knjiga”) in 1969, as part of the programme to mark the 100th anniversary of the birth of this great mathematician. Dragoslav S. Mitrinović initiated the publication of the second issue of these textbooks. A significant contribution was also provided by dr Milorad Bertolino and dr Petar Vasić, who did the editing. In a succinct foreword, the editors gave the most necessary information about the interventions that they carefully carried out.⁶⁵



Cover page of *Differential equations, pt. II, The Collected Works, Book 2* (Digital legacy of Mihailo Petrović)

The third issue of Mihailo Petrović's textbooks was published within the seminal project *The Collected Works of Mihailo Petrović*: book 8 – *Interval Mathematics – Differential Algorithm* and book 9 – *Elliptic Functions – Integration by Use of Series*, prepared by dr Dragan Trifunović and dr Zoran Kadelburg. In addition to the prepared texts of Mihailo Petrović's textbooks, both books contain detailed forewords which systematically and highly illustratively elucidate the most important details concerning the publication of the three textbooks.⁶⁶

The 15 books of *The Collected Works of Mihailo Petrović* were published owing to the Institute for Textbook Publishing and Teaching Aids in Belgrade and its then director dr Dobrosav Bjeletić, as well as thanks to dr Dragan Trifunović who was a tireless guardian of the personality and work of Mihailo Petrović, and the Faculty of Mathematics in Belgrade, Society of Mathematicians of Serbia, and editors and members of the Editorial Board.⁶⁷



Anton Bilimović (1879–1970)



Nikola Saltikov (1872–1961)

MODERNISATION OF CURRICULA

Mihailo Petrović actively worked on upgrading the mathematics curricula at all levels, as illustrated by the following:

At the Fourth International Congress of Mathematicians in Rome from 6 to 11 April 1908⁶⁸, the decision was made to reach – through the comparative analysis of mathematics curricula in participating countries – a single curriculum to be applied in countries wishing to introduce it.⁶⁹ A working group was set up, led by well-known professor of Göttingen University Felix Klein. The group was to establish the International Commission on Mathematical Instruction (ICMI). Mihailo Petrović informed in detail professors of mathematics in Serbia about the events that followed in his inspired text “The International Commission on Mathematical Instruction” published in *Prosvetni glasnik (Educational Gazette)* in 1913⁶⁹. The text shows that the International Commission had 43 full members from 25 countries of Europe, America, Asia and Australia. Our delegate was Mihailo Petrović⁶⁹. The members of the Commission were respectable world mathematicians. Mihailo Petrović informed his colleagues about the Commission’s meetings in Brussels in August 1910, Milan in September 1911, and Cambridge in August 1912.

The International Commission on Mathematical Instruction focused on two issues: a) systematic presentation of mathematics and its disciplines in secondary schools and b) theoretical and practical courses in mathematics for students of physical and natural sciences. Based on voluminous material obtained, Mihailo Petrović forecast the direction of courses in mathematics in the period to come (after 1913). For the sake of comparison with the present-day situation in teaching of mathematics in the country and the world, Petrović's "indications" are worth quoting: "an impetus should be lent to intuition and experiment in lower and medium-level teaching, with the least amount of formalistic elements; a purely practical, utilitarian character will be given to mathematics as the main auxiliary element in expert teaching; while a purely logical direction will be pursued in higher-level teaching, where mathematics is to be taught for the sake of itself, regardless of its role in other areas of knowledge".⁷⁰

Mihailo Petrović then gives a short overview of the most important achievements in the teaching of mathematics in the world. He mentions British mathematics laboratories and textbooks focusing on clarity and intuition, the interest of teachers across Europe in reforming the teaching of mathematics (he mentions Germany and Romania as examples), and expresses his personal enthusiasm that all this took place spontaneously, without any official intervention⁷⁰. He also mentions the partially designed *Bibliography of Mathematical Teaching*, which contained around 2000 titles, classified by content and type of school. Mihailo Petrović sincerely hoped that the reform movement of mathematical teaching would inevitably engulf Serbia and that the educational authorities in Serbia had to show interest in the project. He called upon the teachers of mathematics in Serbia to study the material of around 300 papers printed in 160 issues, and organise a national subcommittee of the ICMI.⁷⁰

Of course, Mihailo Petrović's interest in this area was not limited to mere statements and observations. As was typical of him, he always actively participated in everything, writing papers in which he showed his interests and put forward interesting proposals for the introduction of elements of differential and integral calculus in mathematics curricula in second-level secondary schools.⁷¹

The International Commission on Mathematical Instruction was particularly active until World War I, but went into a decline between the two world wars, exerting no influence on the teaching of mathematics.⁷² This, however, did not prevent Mihailo Petrović and his associates from advancing the teaching of mathematics in Serbia. In 1926, the Mathematical Club was created in Belgrade, led by Anton Bilimović. In 1937, the Yugoslav Mathematical Society was established (the precursor of today's Society of Mathematicians of Serbia), led by Tadija Pejović. The Society gathered around 100 eminent mathematicians and physicists from Belgrade, Zagreb and Ljubljana. Its work was interrupted by World War II.⁷³

POPULARISATION OF MATHEMATICS

Real pedagogues in mathematics differ from those who are not so as they present mathematics by showing all its beauties and numerous applications, trying to popularise mathematics as much as possible, and give it an appropriate, significant place in the educational system. In addition to popularising mathematics among his students, post-graduate students, teachers and associates, Mihailo Petrović played an important role in many other activities aimed at familiarising the public with mathematics and pointing out the great importance of mathematics for civilisational development in general.

As he was an avid reader from his early childhood, Mihailo Petrović knew well what literature meant for the development of a science. As soon as he arrived in the Great School, he began to create a library.⁸⁵ Just before the foundation of the University of Belgrade, libraries were created at other departments as well. The first official data on the Mathematical Library of the Faculty of Philosophy of the Great School date to 1902. Judging by the then established inventory book of the Mathematical Cabinet, the library had over 300 titles. Until the inventory number 100, the library was led by Bogdan Gavrilović, with Mihailo Petrović taking the lead later on. Gavrilović and Petrović obtained mathematical literature through libraries in Belgrade, Paris and Vienna. The notes at the end of each inventory book suggest that they were carefully replenishing the library fund with books on elementary mathematics, applied mathematics, the history and philosophy of mathematics, analytical and higher geometry, higher algebra etc. Between the two wars, Mihailo Petrović and his associates carefully and continuously replenished the library, which was therefore very well equipped. In his lecture notes and textbooks, Mihailo Petrović recommended to his students to use the numerous books and collections from the library in order to deepen and broaden their knowledge. Unfortunately, the mathematical library and many other important documents concerning the work of this great scientist and his associates did not survive the war as they were burnt to ashes in the fire of 18 October 1944.⁷⁴

Mihailo Petrović was also aware of the fact that “high” science had to be presented to pupils and students in a popular way, and that the University and the Academy of Sciences had to be constantly interested in mathematics curricula in primary and secondary schools. The mathematical literature between the two wars thus contains a number of articles that Mika Alas wrote for secondary school students, their



The book *Articles* which contains popular texts of Mihailo Petrović

teachers and fans of mathematics in Serbia.⁷⁵ It is in this regard that we can observe three types of Petrović's contributions: articles – appendices to secondary-school mathematical textbooks, interesting popular texts in journals, and support to the publication of relevant mathematical literature.

Young mathematicians and teachers of mathematics certainly still find topical Petrović's articles: “Real and Illusory Geometric Impossibilities” (1937)^{75,76}, “Wrong Geometric Conclusions from a Carelessly Drawn Picture” (1938)^{75,76}, “Interesting Details in the Application of the Pythagoras' Rule” (1939)^{75,76}, “Indeterminate, Impossible and Incompletely Defined Planimetry Tasks” (1940)^{75,76} and “Falseness of the Eye in Comparing Straight Lines and Surfaces” (1940)^{75,76}. All these articles were published between 1937 and 1940 as annexes to textbooks on mathematics, i.e. geometry for the second, third, fourth, fifth and first year (in order of publication) of secondary school. The authors of these textbooks were Anton Bilimović and Tatomir Anđelić. It is worth noting that Petrović's article “Stereometric Inequalities” was written in 1941 for the geometry textbook for the fourth grade (the textbook was not printed due to the war) and published only in 1953.^{75,76}

Mihailo Petrović wrote similar articles in the most widely read journals for teachers of his time: *Nastavnik (Teacher)*, *Glasnik profesorskog društva (Gazette of Professorial Society)*, *Srpski književni glasnik (Serbian Literary Gazette)*, *Zbornik radova Akademije nauka (Collection of Works of the Academy of Sciences)* or other foreign journals. Particularly interesting are his articles “Absolute and Restrictive Mathematical Impossibilities” (1914)^{75,76}, “A Question from Logarithm Teaching” (1928)^{75,76}, “Squaring the Circle and Angle Trisection Before the Parisian Academy of Sciences” (1928)^{75,76}, “Mathematicians' Errors” (1933)^{75,76}, “Squaring the Circle” (1938)^{75,76}.

We wish to particularly highlight the article “About the Dependencies Among Magnitudes in Tasks”, published by Petrović in 1932 in *Matematički list za srednju školu (Mathematical Paper for Secondary School)* (No 3–4, p. 37–44)⁷⁷. The article is rather interesting – diminished caution in formulating tasks often leads to errors arising from the lack of knowledge about dependencies among magnitudes in tasks, so that the characteristics of mathematical objects are sought, which, with the defined characteristics, in fact do not exist.⁷⁷ It is also worth noting that the owner and editor of *Matematički list* was one of Petrović's students and closest associates – Jovan Karamata, and that the journal had a Yugoslav editorial board.⁷⁸

All the stated, and other non-stated articles⁷⁹, were written in a simple, popular, comprehensible language and with an acceptable style. The themes of all enumerated articles were obviously very useful, very carefully chosen, highly interesting and topical, and largely interesting still today as part of the general mathematical culture of every young person. We shall give only one, but a very characteristic example. If some journalists had read Petrović's texts about the squaring of the circle or angle trisection, as his students did, it would have never crossed their minds to write in our daily with high circulation the truly “sensational” news: “A professor from Užice defeats Gauss and Descartes with his compasses and ruler”.⁸⁰

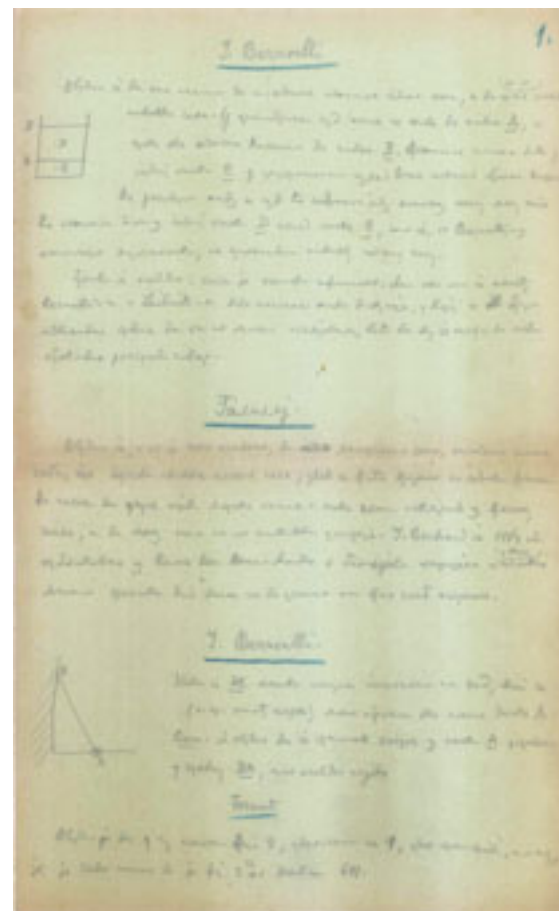
MIHAILO PETROVIĆ'S OTHER PEDAGOGICAL CONTRIBUTIONS

Mihailo Petrović was also active in other areas of pedagogical research. It seems he was a man on whom the then Ministry of Education and Church Affairs could always count, regardless of whether he was entrusted with tasks in Belgrade or the interior.

For a number of years, he was a member of the professorial exams committee. The first time, already in 1895, he and Bogdan Gavrilović had five candidates⁸¹. Somewhat later, in 1913, Mihailo Petrović was elected a member of the standing professorial exams committee with a four-year mandate. The importance attached to professorial exams at the time in Serbia is best seen in the composition of the committee, which also consisted of Pavle Popović, Jovan Skerlić, Aleksandar Belić, Jovan Cvijić, Bogdan Gavrilović, Sima Lozanić and other distinguished professors of the University⁸¹.

Today, when many doubt the validity of the future national maturity exam, highly relevant is the fact that Mihailo Petrović was also a member of committees supervising final exams in Serbian gymnasiums such as the First Belgrade Gymnasium (1896, 1898)⁸¹, Gymnasium in Kragujevac (1900, 1906)⁸¹, Gymnasium in Niš (1902)⁸¹, Gymnasium in Jagodina (1906)⁸¹, Second Belgrade Gymnasium (1908)⁸¹. In terms of our present-day national maturity exam, it must be emphasised that back in 1902 Mihailo Petrović, owing to his experience in maturity exams in gymnasiums (which he praised), believed that only those students who passed the maturity exam could enrol in the University, as such exam was the best reflection of their actual knowledge⁸¹.

Mihailo Petrović was a member of the Main Education Council of the Ministry of Education on several occasions (1896, 1898, 1900)⁸¹, and served as its president in the 1909–1910 period⁸¹. As the Council's member and president, he could more decisively influence the development of the teaching of mathematics in secondary schools (curricula, textbook reviews etc.).



Handwritten notes and comments of Mihailo Petrović for one of his works (Society "Adligat")

Mihailo Petrović was occasionally appointed the supervisor in secondary schools (academic years of 1897/98, 1909/10 – the Third Belgrade Gymnasium)⁸¹, which seems to have been a good practice. This method could be implemented today as well since the engagement of the most renowned university professors could significantly improve the cooperation between secondary schools and universities, and enable continuous monitoring, at least partial supervision of the work of schools, better understanding of their work, and greater possibility to upgrade all activities in schools.

It is worth noting that Mihailo Petrović also discharged several important functions at the University of Belgrade. He was the dean of the Faculty of Philosophy from 1908 to 1909⁸², and its vice-dean three times – from 1909 to 1913⁸². He was proposed to be the rector of the University of Belgrade several times, but he refused these offers for the reasons of personal and political nature (although he never dealt with politics), which are not related to teaching or scientific factors⁸².

The pedagogical contributions of Mihailo Petrović can be elaborated in even more detail since he exerted an undoubtable influence on the establishment of the Council of Yugoslav Students of Mathematics (1927)⁸², formation of general staff officers in the area of code work (he wrote the textbook *Cryptography* in 1928)⁸² and the establishment of the Department for Geometry (1930).⁸²

At the end of this chapter, it should be noted that Petrović's pedagogical achievements were not related to Serbia only as his teaching influence went beyond the borders of our country. The most typical example is that from 1928, when Petrović was a guest lecturer during a summer term at Paris University, holding a one-term course on mathematical spectres.⁸³

INFLUENCE OF MIHAILO PETROVIĆ'S PEDAGOGICAL WORK ON PRESENT-DAY TEACHING CIRCUMSTANCES IN SERBIA

The influence of Mihailo Petrović's scientific work on the present-day teaching practice in Serbia is doubtless significant. Petrović's greatest pedagogical achievement were his students. After World War II, his closest associates and followers did a lot for the development of mathematical science and teaching in Serbia and Yugoslavia. Several hundred mathematicians who are direct successors of Mihailo Petrović work today in Serbia, at universities, institutes and schools, carrying out the most important teaching and scientific duties in the Serbian Academy of Sciences and Arts, Mathematical Institute, Society of Mathematicians of Serbia, universities and faculties, Mathematical Gymnasium and other institutions. Also important for present-day mathematics in Serbia is the genealogical tree of dr Đuro Kurepa, which contains around 160 mathematical successors.

Worth mentioning is that all the above institutions and their present-day activity were designed by students and associates of Mihailo Petrović or their students (Jovan Karamata, Anton Bilimović, Tadija Pejović, Radivoj Kašanin, Miodrag Tomić, Dragoslav Mitrinović, Vojin Dajović, Milica Ilić Dajović, Slaviša Prešić etc.).

The analysis of Mihailo Petrović's teaching ideas⁸⁴ from his 44-year professorship at the University of Belgrade suggests that there is almost no segment where his exceptional work was not continued, as well as that, quite naturally, these ideas were further developed and improved in numerous fields. The table below contains one of possible comparisons only for Petrović's most important teaching activities:

Mathematics Department of Belgrade University	Faculty of Mathematics in Belgrade, mathematical departments and institutes at faculties of science and mathematics in other university centres
Mathematical Institute of Belgrade University	Mathematical Institute of the Serbian Academy of Sciences and Arts
Mathematical Library	Library of the Faculty of Mathematics and the Faculty of Science and Mathematics, University Library, National Library, Library of the Academy of Sciences and Arts, Library of the Society of Mathematicians of Serbia
Mathematical Club, later Yugoslav Mathematical Society	Society of Mathematicians of Serbia
Journal <i>Publication mathematiques de l'Université de Belgrade</i>	Journal <i>Publications de l'Institut Mathématique (nouvelle serie)</i>
<i>Matematički list za učenike srednjih škola</i>	<i>Matematički list za učenike srednjih škola</i> and <i>Tangenta</i> for secondary schools
Popularisation of mathematics	Society of Mathematicians of Serbia, Mathematical Society "Arhimedes", School for Lovers of Mathematics "Integral"

The present-day teaching of mathematics in Serbia contains areas not existing at the time of Mihailo Petrović, but it is worth noting that these ideas are mainly developed by his successors. These include: seminars for teachers and teaching conferences, mathematics teaching journals, publishing activity popularising mathematics, work with gifted students, mathematical competitions, summer and winter schools of young mathematicians etc.

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The above sections of this paper contain a lot of information about the teaching work of Mihailo Petrović. Each footnote suggests the enormous effort that Mihailo Petrović invested in the education of young people and their familiarisation with science. At the same time, each of those footnotes contains a great story as it sheds light on a concrete event, endeavour or book, article, study, and life story that Mihailo Petrović left behind himself. Analysing the voluminous bibliography of his works⁸⁵, which contains around 400 bibliographical units (and numerous excerpts, overviews, quotations) and rich general literature⁸⁶ about Petrović, one can see that materials concerning the teaching of mathematics are so comprehensive that they can be used to write a number of papers, studies and even doctoral dissertations.

It is indisputable that each day of his professional career Mihailo Petrović worked on the mathematical enlightenment of the milieu that he lived in and that his teaching ideas and activities relating to the popularisation of mathematics have strong implications for the present-day teaching practice in the field of mathematics in Serbia.

Mihailo Petrović was a brilliant lecturer. He was correct towards his students. He respected their effort and supported their constructive proposals. Just as he was strict towards himself, he was strict at exams, but created an atmosphere of immediacy at his courses⁸⁷.

However, it would be unjust to observe Mihailo Petrović Alas only as a teacher and scientist as over a half a century he shaped the social life in Serbia of his time. Let us therefore at least briefly illuminate the social aspect of his personality and his unusual versatility. Petrović was at the same time both a sociable and withdrawn person. Although he was affable, he never imposed himself or showed off (he shunned great publicity). He was composed, somewhat shy⁸⁷, moderate (apart from work, travels, music and fishing) and modest⁸⁷. He was obviously very pleasant, humorous, communicative, systematic⁸⁷ and disciplined. His general culture was at the highest level. The abundant preserved correspondence of Mihailo Petrović testifies to a highly accurate, well-organised and polite man, who did not leave his professorship work for a single day without informing beforehand his superiors and associates. He would leave no one's letter unanswered. Mika Alas was certainly a unique personality also judging by the fact that he socialised with people from the widest social strata⁸⁷ – fishermen on the Sava and Danube rivers, Roma musicians in Savamala, visitors and owners of numerous Belgrade taverns, his school friends and professors, students and associates, world-class scientists and members of the royal family. The extent to which his students, associates, acquaintances and friends liked him is best attested by numerous anecdotes from the life of old Belgrade, and even songs in his honour⁸⁸. Mihailo Petrović was also a humane person. His biography illuminates many places and events

showing that he often gave up on his pay, helped the poor and feeble, gave away the caught fish, or helped his friends to make ends meet.

Finally, let us also say that the life and work of Mihailo Petrović Alas, his patriotism and exceptionally rich work are an example of how to help one's nation and its future, as well as a very good chance for young people (those whom we educate and reveal to them the secrets of mathematical science) to be inspired by the work of Mihailo Petrović Alas, a wondrously original personality and one of the most important figures in the history of the Serbian people.

REFERENCES

- [1] Драган Трифуновић, *Лейхойис живоџа и рада Михаила Пеџровића*, Српска академија наука, Београд, 1969, VIII + 361.
- [2] Михаило Петровић, *Елийџичке функције* (друго издање), Научна књига, Београд, 1969, VIII + 113.
- [3] Михаило Петровић, *Чланци* Друштво математичара и физичара Народне републике Србије, Научна књига, Београд 1949, V + 110.
- [4] Сабрана дела Михаила Петровића – књига 9, *Елийџичке функције, Инџеџрација џомоћу редова*, Завод за уџбенике Београд, 1997, 323.
- [5] Сабрана дела Михаила Петровића – књига 8, *Инџервална маџемаџика, Диференџијални алџоритџам*, Завод за уџбенике, Београд, 454.
- [6] Михаило Петровић, *Рачунање са бројним размаџима* (друго издање), Издавачко предузеће „Научна књига“, Београд, 1969, VIII + 169.
- [7] Сабрана дела Михаила Петровића – књига 10, *Чланци, Сџудије*, Завод за уџбенике, Београд, 410.
- [8] Владимир Мићих, Зоран Каделбург, Војислав Андрић и други, *Седамдесетџ џодина Друшџива маџемаџичара Србије*, Друштво математичара Србије, Београд, 2018, 137.
- [9] Група аутора, *Михаило Пеџровић Алас – родонаџелник срџске маџемаџичке школе (каџалоџ уз изложбу)*, Српска академија наука, Београд, 2018, 159.



MIHAILO PETROVIĆ
IN PHILOSOPHY, LITERATURE
AND PUBLIC LIFE

