DOI https://doi.org/10.2298/MUZ1824051M
UDC 530.145.6:785/789

The Quantum Vibes of Atoms and Ichthyosaurs*

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Received: 24 March 2018
Accepted: 7 May 2018
Original scientific paper

Abstract

The quantum mechanical description of microscopic phenomena treats minuscule particles as waves and explains why atoms and molecules absorb and emit radiation at particular frequencies. This article reviews the physical theory of waves and discusses similarities between atoms and musical instruments. In particular, it describes how we may identify new musical scales and harmonies and play atomic music by translating and scaling the frequencies in the atomic world to the audible spectrum.

Keywords: waves, atoms, musical instruments, atomic music, audible spectrum

Quantum Mechanics appeared in the mid-1920s as a complete theoretical framework for microscopic dynamics. At the time, scientists had experimentally established that different atoms absorb and emit light with very distinct frequency and wavelength, and the Danish physicist Niels Bohr had incorporated the recent discoveries of the electron and the atomic nucleus in a theory of matter based on Newton's classical mechanics. Bohr had thus in 1913 invoked the idea that the electron orbits the nucleus in the same manner as the planets orbit the Sun, but the motion must be restricted to preferred orbits with definite energies, E_1, E_2 etc. (Bohr 1913: 1). Light is emitted when the electron jumps between such orbits number n and m, and it has a definite energy, $E = hf = E_n - E_m$, and a corresponding optical frequency f, given by $E = hf$, where h denotes Max Planck's fundamental constant. The restriction to special orbitals and the jumps between them were postulated by Bohr and had no basis in existing physical laws, but they led to a very accurate formula for the energies and optical frequencies for the hydrogen atom. Despite great efforts by many physicists, however, no accurate theory could be derived for atoms other than hydrogen.

The French physicist Louis de Broglie then suggested that one should describe the electrons as waves rather than as particles. The idea seems radical, but it may have

* This article was written for the project *Quantum Music*, co-financed by the EACEA within the programme *Creative Europe* (559695-CREA-1-2015-1-RS-CULT-COOP1).

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drawn a glimpse of intuition from the physical theory of music and musical instruments! Indeed, many physical objects oscillate at very regular and well-defined frequencies: sound in organ pipes and wind instruments, strings and membranes. These phenomena are not described by a single moving object or by the motion of the individual atoms that constitute the physical objects, but rather by the collective motion of the whole object. The motion of a vibrating string is thus represented by as a continuous deformation from equilibrium along the string. If the string is bent in one location, the tension pulling towards its ends straightens the string, but it may continue its motion and soon after bend to the other side.

The reader is invited to recall how a skipping rope may be set in oscillatory motion at specific frequencies, where the midpoint of the rope oscillates naturally up and down at a slight motion made by the hand holding the rope. Moving the hand at a different frequency produces only a little motion, but at twice the frequency, one induces another, figure-8 wave pattern with the middle of the rope at rest, while the two opposite halves oscillate up and down, opposite to each other. This phenomenon is governed by Newton's mechanical laws of motion, applied to every segment along the rope and taking into account the forces they exert on each other by means of a so-called wave equation. Denoting the frequency of the simple up and down motion as f_1, one finds a whole sequence of other regular solutions with frequencies $f_n = n{\cdot}f_1$, where $n=1, 2, 3, \ldots$ labels the frequency and also counts the number of wiggles along the string as it oscillates. When a guitar string is plucked, the shape of the string does not match any of these simple wave patterns, but it may, indeed be composed as a superposition, i.e., a sum of the different patterns with different pre-factors. As a result, the motion of the string is decomposed as the sum of these different components, representing the overtones associated with the tone played. Adding the possibility of changing the shape and frequency of the vibration by pressing the string against frets on the fingerboard, we obtain the possibility of playing a variety of tones and harmonies, but the frequencies are always given by the natural wavy patterns of motion of the string.

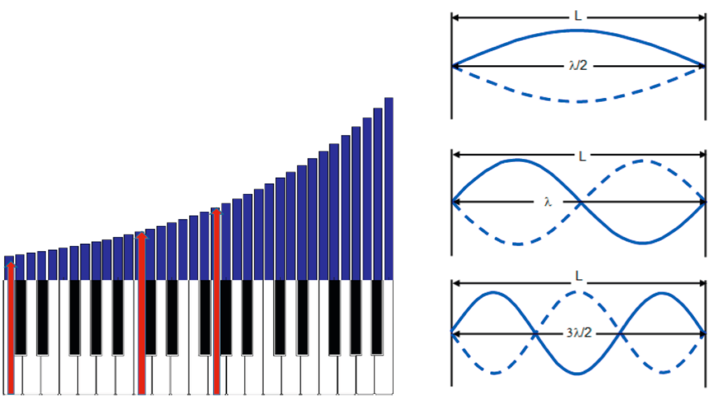

Figure 1. *Three octaves on a piano contain tones with frequencies, shown as the height of the vertical bars. When the low C is played, the corresponding string in the piano is set in motion, but not only a simple up-and down motion along the length of the string. Higher harmonics, corresponding to the figure 8 in the middle panel and the three wiggles in the lower panel on the right are also played by the same string. These resonate well with the tones played by the higher C and G keys on the instrument, because they oscillate at the same frequencies.*

In wind instruments, the compression of the air is described by a similar mathematical equation to the excursions of the vibrating string, and it shows a similar variation along the length of the instrument, which thus dictates similar tones and overtones. On account of the construction of the clarinet being closed in one and open in the other end, the progression of overtones differs from the string and contains frequencies, $f_n = n \cdot f_1$, where $n=1, 3, 5$ … explores only the odd numbers (the octave is missing), while the saxophone, owing to its conical expansion at the end of the instrument has the same progression of tones as the vibrating string. These mathematical properties have consequences for the sound of the instruments and for the way they are played, and it is likely that the mathematical rules of natural instruments have influenced our taste for harmonies, as the interval from f_1 to $2 \cdot f_1$ is nothing but an octave while the interval to $3 \cdot f_1$ brings us to the natural fifth (within the next octave), see Figure 1.

Returning to 1924, and observing that atomic systems lead to the emission of light – not sound – at regular frequencies, it is perhaps not so strange an idea for Louis de Broglie to suggest that whatever happens inside the microscopic atomic particles, it may be described by the theory of waves. Equipped with solid experience with the mathematical equations that describe vibrating strings, sound, and radio signals, the Austrian physicist Erwin Schrödinger, managed to identify a suitable wave equation in late 1925 (Schrödinger 1926: 1049). In his equation, Schrödinger incorporated the known force between the electron and the nucleus, but he described the motion of the electron as if it were a wave delocalized in space, rather than as a particle following an orbit with time. Schrödinger's equation was a success and it has turned out to apply across all microscopic systems in physics. Whenever we solve the equation, for the constituents of atoms, molecules, nuclei and solid-state materials, we find oscillating, wavy patterns of motion, and depending on the forces at play, we find the allowed motional energies E_n and E_m of the system, which subsequently explain which energies may be emitted as light with a frequency obeying, $hf = E_n - E_m$.

Solving the Schrödinger Equation for the electron in the hydrogen atom produces wave patterns in the shape of rings and clover patterns with multiple leaves, and they show considerable similarity with water waves and also with the rapid motional pattern of the surface of, e.g., a guitar or a percussion instrument, when they are played by the musician. In particular, the long wavelengths and large structures are associated with the low frequencies while fine oscillation patterns match the higher frequencies, just as in musical instruments.

In combination with computer calculations, the trained physicist uses his or her visual intuition about waves to predict the outcome of experiments and to suggest methods to control physical and chemical processes. The same intuition and combination of insights are at the heart of the collaboration between scientists and musicians in the quantum music project: the physicist can provide frequencies originating from different physical systems and processes, while the composers, instrument builders and musicians may produce an audible and artistic rendering of the same phenomena.

Let us briefly describe two successful projects and their main outcomes: *BEC Music* for cello and symphony etta and *Super Position (Many Worlds)* for two pianos, both by contemporary Danish composer Kim Helweg.

A Bose-Einstein Condensate (BEC) is a form of matter where many atoms, formed at high temperatures, gradually cool down and as they condense, as water vapour condenses on a cold surface, their individual waves coalesce into a single macroscopically populated wave. The phenomenon was predicted in the 1920s but was observed for the first time only in 1995 (see a review of the properties of BEC in Anglin and Ketterle 2002: 211). The condensate has characteristic oscillation frequencies, and when interactions dominate the motion in one spatial dimension, we obtain an approximate formula for its frequencies $f_n = \sqrt{n(n-1)} \cdot f_1$, with $n=2,3$ … The lowest frequency f_1 in current laboratory experiments is tens or hundreds of Hz, and is hence in the audible range, and the wave indeed describes a variation in pressure and density of a real gas, *i.e.*, a real sound. The experiments, however, have to isolate the BEC from disturbances by the hot air in the atmosphere, and the recording of the motion in the lab is therefore done with a camera rather than with a microphone (Kristensen et al., 2017). By external forces, but also by the mere observation of the system, the experimentalist can induce a slushing motion of the atoms (Wade, Sherson and Mølmer 2015), and there are prospects of using such motion to sense gravity, and acceleration effects. In the *BEC Music* project, composer Kim Helweg used inspiration from the physical process of cooling and coalescence of the many individual atoms in the condensate and the correlated motion in the system, and he explored the sounds and harmonies offered by the BEC and other condensed matter quantum systems.

The piano suite *Superposition: Many Worlds* composed by Kim Helweg for the Quantum Music project explores the optical spectra of the hydrogen atom. The atom emits and absorbs radiation associated with quantum jumps between different wave solutions of the Schrödinger Equation. The actual frequencies range up to 15 digit numbers of oscillations per second, corresponding to the near ultraviolet spectrum of light, and the emission is neither of the acoustic type nor anywhere near the audible range. The composer made a choice to compose pieces separately for each of the so-called spectral series, named after their early scientific discoverers, and corresponding to the quantum jumps ending on specific final states.

Lyman series:	$h \cdot f_n = E_n - E_1$, $n= 2, 3, \dots$,	ending in the lowest (ground) state of the atom
Balmer series:	$h \cdot f_n = E_n - E_2$, $n= 3, 4, \dots$,	ending in the first excited state of the atom
Paschen series:	$h \cdot f_n = E_n - E_3$, $n= 4, 5, \dots$,	ending in the second excited state of the atom

Each of these series represents a spectrum of growing frequencies, and increase by a constant factor given by a simple formal expression, that we can write in short mathematical form as $E_n = -1/n^2$. Note that as n becomes larger and larger, the values of E_n all become vanishing small, and hence the frequencies in the different series converge to the highest achievable values – $E_1 = 1$, $- E_2=0.25$, $-E_3 = 0.111$, … (all multiplied with a suitable basis frequency). See the appendix for tables with the frequencies.

For the adaption of each series to music instruments, we allow multiplication of the atomic frequencies by a constant value, and we subsequently find the actual tone on the piano with the closest value of the frequency to the desired atomic sound. This is illustrated by the matching, in Figure 2, of the vertical position of the red bars indicating atomic frequencies, and the height of the blue vertical bars representing frequencies of the tones of the keyboard. On string or wind instruments, the musician can address quarter-tones or even finer divisions of the intervals and with digital technologies one may play the atomic frequencies, shifted to the audible range without rounding off to the nearest piano key. We may also employ digital technologies to ensure a composition of overtones belonging to the desired spectra, rather than having recourse to the integer progression of overtone frequencies dictated by the strings in the real piano.

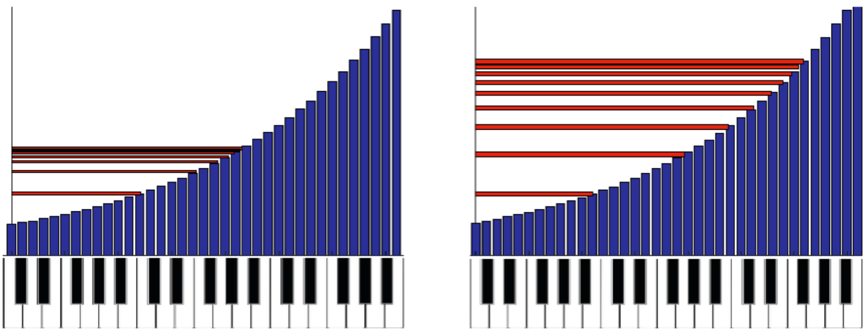

Figure 2. *The frequencies of the atomic spectrum of hydrogen are sorted in series. The second, so-called Balmer series, shown on the left, has frequencies separated by a large gap and converging to the same value (shown as the height of the red bars in the figure). The sixth series, shown on the right, spans a wider range with more even frequency gaps. The actual values of the frequencies must be scaled by a very large number to go from optical to audible frequencies.*

Since the dawn of quantum theory, it has been a puzzle for physicists to understand what is actually going on at the microscopic level. The wave equation by Schrödinger not only introduces a new physical law and new phenomena, but it forces us to address the very concept of physical reality in a new way. The description of physical particles as waves suggests that they are delocalized in space, but still, in experiments, we always find them at definite but random locations when we look for them. So are they already at the location, where we find them, or are they really at several places at the same time until we look for them?

When quantum theory is applied to several particles, a new strange feature appears: entanglement. According to the wave description, two particles are described by a wavefunction of two coordinates. If one particle is detected, the wavefunction of its entangled partners may change abruptly. Neither Einstein nor Schrödinger were willing to accept this kind of "spooky action" (Einstein, Podolsky and Rosen 1935: 777), and Schrödinger famously proposed that if one particle is an atom and the other is a cat, the strange wave properties of the atom should eventually also apply to the cat. Could a cat inside a box be both dead and alive if an atomic decay were controlling a poison mechanism within the box? The entanglement goes even further such that even millions of atoms and molecules may share a single energy quantum such that it is stored in any one of them at the same time (a property that may be used for quantum memory storage; see Tordrup and Mølmer 2008). In 1952, Schrödinger was still occupied by the foundational questions of the theory and suggested that it should never be applied to single objects but always be thought of as a statistical description of many objects: "it is fair to state that we are not experimenting with single particles any more than we can raise Ichthyosauria in the Zoo" (Schrödinger 1952: 109). By this argument, we should only be concerned with the final count after carrying out the cat experiment many times: out of the many cats, some will be dead and some will be alive, but no cat is both dead and alive.

Experiments have become possible today that it was not possible for Schrödinger to predict in 1952 and today we can perform experiments with single atoms and other quantum systems; one of the fascinating challenges of current research is to control and manipulate the motion of atoms that evolve as waves. We developed a smartphone game to let public players try their luck with quantum waves, and indeed they found useful solutions for experiments (Sørensen et al. 2016: 210). A whole new quantum information theory is currently being built upon the wave, randomness and entanglement features which, by bringing the Ichthyosaur alive, may offer 100% security against eavesdropping on communication channels (Bennet, Brassard and Ekert 1992: 50), very precise detection and sensing (Giovannetti, Lloyd and Maccone 2006) and exponentially increased performance of computers (Nielsen and Chuang 2010).

Discussions among scientists and music experts about the artistic rendering and exploitation of these more profound effects: the correlations between different components, the randomness and the non-trivial consequence of observations constitute a challenging next step for the quantum music project. So far, we have only scratched the surface of what may result from bringing together science and music experts. It has been important for the project not to make music that is bound to illustrate or explain physical effects, but rather to exchange knowledge and use the physics material in its bare form as if we are being offered a new instrument with new properties.

At a seminar about the Quantum Music project in 2016, the author was asked the question: "How can we be sure that it will sound good?" In a somewhat cavalier fashion, he answered that this was entirely the responsibility of the composer and the musicians. Without them, how could we ever be sure that music would sound good?

Appendix

In this technical appendix, I provide some details on the numerical values of frequencies, and the choices available when we want to play on the atoms.

In the main text I discussed the mechanism of overtones of the oscillating string at 2 and 3 times the frequency of the main tone played, respectively – see Figure 1. On a tempered piano, all semitone intervals represent the same increase of frequency by the factor 1.059463, such that 12 half tones yield a factor of 2 increase in frequency: an octave. With the tempered scale, frequencies of all tones follow a simple mathematical progression, and setting the A above the middle G to the concert pitch A440, i.e., a frequency of 440 Hz, defines the frequencies sounded by all keys on the piano. With 12 semitones to an octave, 7 semitones correspond to an increase in frequency of 1.498, which is almost equal to 1.5. This makes the seventh semitone interval sound close to the natural fifth (the ratio 3/2 between the overtone frequencies of the string). We could have imagined a division of the scale in any other number of "semitones", say 19 or 27, but 12 is a particularly good choice: we would need 41 semitones on a tempered instrument to obtain an interval closer to the natural fifth!

The atomic spectra yield optical frequencies of light that we can multiply by a suitable number to get values in the range of tens to thousands of Hz (oscillations per second). For the hydrogen atom, Niels Bohr established the existence of different series of frequencies given by simple mathematical expressions,

Lyman series:
Formula: $f_n = f_0\,(1/1^2 - 1/n^2) = f_0\,(1 - 1/n^2)$, n= 2, 3, ...
Values of terms in brackets: ( ) = 0.7500 0.8889 0.9375 0.9600 0.9722 ...
Balmer series:
Formula: $f_n = f_0\,(1/2^2 - 1/n^2) = f_0\,(1/4 - 1/n^2)$, n= 3, 4, ...
Values of terms in brackets: ( ) = 0.1389 0.1875 0.2100 0.2222 0.2296 ...
Paschen series:
Formula : $f_n = f_0\,(1/3^2 - 1/n^2) = f_0\,(1/9 - 1/n^2)$, n= 4, 5, ...
Values of terms in brackets: ( ) = 0.0486 0.0711 0.0833 0.0907 0.0955 ...
8th series:
Formula : $f_n = f_0\,(1/8^2 - 1/n^2) = f_0\,(1/64 - 1/n^2)$, n= 9, 10, ...
Values of terms in brackets: ( ) = 0.0033 0.0056 0.0074 0.0087 0.0097 ...

The prefactor f_0 is the same in all the atomic formulas and has the value $f_0 = 3.28 \times 10^{15}$ *Hz* for the very high optical frequencies emitted by the hydrogen atom (10^{15} is a number written as 1 followed by 15 zeros). The terms in the parentheses are readily calculated, and the first few numbers are listed above, e.g., the first term in the Lyman series: $(1\text{-}1/2^2) = (1\text{-}0.25) = 0.75$, and the third term in the Paschen series $(1/9\text{-}1/6^2) = (1/9\text{-}1/36) = 0.0833$.

To reach the audible range, we keep the numbers tabulated above, but we assume a different value for f_0. The Lyman and Balmer series show a clustering of values near *1.00* and *0.25*, respectively, and many tones fall within the same frequency interval of the tempered scale. The composer therefore stretched the intervals by a factor to obtain more discernible tones. As the higher series lead to much smaller values, but also to progressions over more regularly increasing frequencies, they span many different tones and a similar stretching was not deemed necessary. For details of the composer's choices and elaboration of the material, see Kim Helweg's article in the present issue (Helweg 2018).

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Клаус Молмер

Квантне вибрације атома и ихтиосауруса

(Сажетак)

Квантно-механички опис микроскопских феномена третира мајушне честице као таласе и објашњава зашто атоми и молекули апсорбују и емитују радијацију на одређеним фреквенцијама. У овом чланку дајем преглед физичке теорије таласа и разматрам сличности између атома и музичких инструмената. Посебно објашњавам како можемо идентификовати нове музичке скале и хармоније и изводити "музику атома" путем превођења фреквенција из света атома у спектар звукова које људско ухо може да чује.

Данас је могуће вршити експерименте са појединачним атомима и другим квантним системима, а један од најфасцинантнијих изазова тренутних истраживања јесте контролисање и манипулација кретања атома који еволуирају у таласе. Између осталог, развили смо игрицу за паметне телефоне која омогућава најширим слојевима корисника да се поиграју са квантним таласима, чиме постају саучесници у експериментима (Sørensen et al. 2016: 210). Данас се изграђује читава нова квантна информациона теорија заснована на карактеристикама таласа, произвољности и умрежености, која може омогућити стопроцентну заштиту против прислушкивања комуникационих канала (Bennet, Brassard and Ekert 1992: 50), затим, врло прецизно детектовање и осетљивост (Giovannetti, Lloyd and Maccone 2006) и веома побољшане перформансе компјутера (Nielsen and Chuang 2010).

Разговори вођени између научника и музичара у вези са уметничком експлоатацијом корелација између различитих компоненти и не-тривијалних последица посматрања представљају следећи изазован корак за пројекат квантне музике. До сада смо само загребали површину поља могућности које нуди спој науке и музике. За овај прелиминарни пројекат било је важно да се не ствара музика која би само илустровала или објашњавала физичке ефекте, већ да се размене искуства и да се физички материјал употреби у својој најосновнијој форми – као да нам је дат нови инструмент са новим могућностима.

Кључне речи: таласи, атоми, музички инструменти, атомска музика, чујни спектар