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The Musical Mach–Zehnder Interferometer*

Andrew J. P. Garner[1]
Centre for Quantum Technologies, National University of Singapore - Institute for Quantum Optics and Quantum Information, Vienna, Austrian Academy of Sciences

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Abstract

The phenomenon of *interference* lies at the heart of quantum physics, and is responsible for many of the unusual aspects of quantum behaviour that deviate from our everyday expectations. Though classical physics allows for waves (e.g. of sound) to interfere, quantum theory allows for interference effects also to affect *single particles*. One device that demonstrates this experimentally is the *Mach–Zehnder interferometer:* here a single particle (e.g. a photon) travels down one of two possible paths, and quantum interference between the two paths affects the final position where the particle arrives. In this article, I propose a mechanism to musically demonstrate quantum single-particle interference: the musical Mach–Zehnder interferometer. This new *quantum musical instrument* makes use of two independently operated electronic keyboards, whose outputs are interfered according to the rules of the Mach–Zehnder interferometer. I discuss the musical possibilities this instrument enables, and outline a method to construct it via software simulation.

Keywords: quantum physics, interference, Mach–Zehnder interferometer, new musical instrument

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1 music@ajpgarner.co.uk

Scientific Background

Sound travels through the air in waves of changing pressure. Suppose two monitor loudspeakers are generating pure tones of sound in a room with minimal acoustics. At some points in the room, the pressure waves arriving from each speaker line up such that both waves are at their maximum pressure. Here, the effect of the waves add together, and a louder sound is heard. There are also points in the room where the pressure of the wave from one speaker is at its maximum, and from the other speaker is at its minimum. In these places, the waves perfectly cancel out and no tone is heard at all.

This phenomenon is known as *interference*, and is one of the most important aspects of wave physics. When the two waves arrive together, they are said to be *in phase*, and this results in *constructive interference* – the waves add together to produce larger changes in pressure. On the other hand, when the two waves arrive to change the air in completely opposite directions, they are said to be *out of phase*, and this results in *destructive interference* – the overall effect is that no sound at all is heard. Whether the waves are in or out of phase depends on the relative lengths between the listener and each monitor.[2]

The mathematics of quantum theory (see, for example, Dirac 1930; Griffiths 1995) behave in much the same way – but with the added twist that the waves themselves are trickier to interpret. The Schrödinger equation, which lies at the core of quantum mechanics, itself is an equation for wave motion. Instead of the waves being disturbances of air pressure, or changes in electric field, they are waves of *probability*. Unlike classical mechanics, where every object (no matter how small) has a well-defined position and speed, this intuition does not extend to quantum mechanics. A quantum particle's position is only well defined when it is measured. The chance of finding a particle in a particular location is governed by these probability waves.[3]

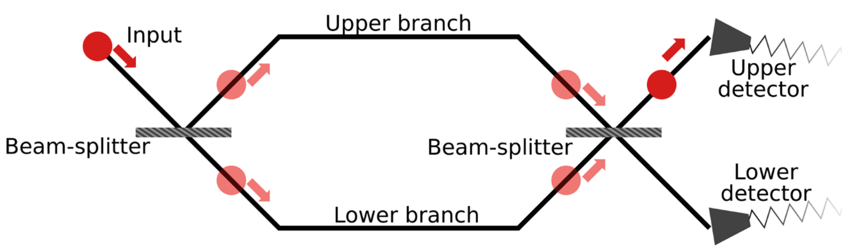

Figure 1. The Mach–Zehnder Interferometer. *A particle enters from the left. The first beam-splitter places the particle into a quantum superposition, such that it travels down the upper and lower branches at the same time. At the final beam-splitter, the paths recombine and if the particle travels to the upper detector with certainty.*

2 The same phenomenon occurs when using two microphones to simultaneously record an audio source. If the signals are in phase, constructive interference results in a "punchy" sound of the mixed signal. If they are out of phase, destructive interference thins the sound, as the two recordings cancel each other out.

3 The absolute square of the value of the wave-function gives the probability density.

One of the simplest physical set-ups that can demonstrate interference is the *Mach–Zehnder Interferometer* (MZI) (Zehnder 1891; Mach 1892) illustrated in Fig. 1. First consider the classical (non-quantum) version of this device, where a beam of light is input (light is itself a wave of electric and magnetic fields). At the heart of this device are *coherent beam-splitters* – semi-reflective mirrors that divide the light into separate beams that travel along the two branches of the interferometer. When the beams recombine at the second beam-splitter, depending on whether the light arrives in or out of phase, it will be steered into one detector or another. By choosing the length of the branches carefully, it is possible to guarantee that *all* the light goes into one detector, or with a different choice of length, that all the light goes into the other.

Interference behaviour may be contrasted to *incoherent mixing*. In this case, both beam- splitters randomly direct the light such that 50% goes one way, and 50% the other. Here, no matter what the relative length of the two branches, one always sees the same amount of light arriving at each detector (an equal amount) – phase has no effect on incoherent mixing.

The quantum version of the interferometer manifests when we consider a single particle travelling through it. Suppose one sends in a photon (a single particle of light). At the first beam-splitter, classical physics tells us that the particle should go randomly down one of the branches. However, quantumly, this is not what happens. Rather, the photon's behaviour here cannot be described like a particle, but must be treated instead as a wave of probability. After the first beam-splitter, these probability waves are equally divided, such that if one tried to measure the photon, it would be found on either side with equal probability. However, the particle itself is not on either side. It is now travelling down *both branches at the same time* – a phenomenon known as *quantum superposition*. This state of superposition is maintained so long as one does not measure which branch the particle is in.

When these waves of quantum probability meet again at the second beam-splitter, interference occurs. By controlling the relative length of the paths such that the probability waves arrive in phase, one can guarantee that the particle always ends up in one detector with certainty. (Likewise, engineering the path lengths so that the waves arrive out of phase ensure that the particle always arrives in the other detector with certainty). This is very counter-intuitive when considering the behaviour of classical particles: where a beam-splitter that equally divides the beam should always result in the particle being found equally likely in either detector. This difference in the behaviour between quantum and classical is the difference between coherent interference and incoherent mixing.

The Mach–Zehnder interferometer is intrinsically related to Deutsch's algorithm (Deutsch 1985; Deutsch and Jozsa 1992; Cleve et al. 1998) – one of the earliest algorithms to demonstrate the advantage of quantum computers over their classical counterparts. Here, one considers a function *f* that takes a binary input (0 or 1) and returns a binary output. Either both inputs 0 and 1 return the same output and the function is *fixed*, or each input results in a different output and the function is *balanced*. To determine whether the function is fixed or balanced using a classical computer, one would have to evaluate function with both inputs in turn, and then compare the results. However, using a quantum computer, it is possible to ask both inputs *at the same time* and, using interference, recover whether the answers are the same or different in a single step.

This can be physically implemented using the Mach-Zehnder interferometer, described above. Suppose on each branch of the interferometer, there is a sealed box that either allows the particle to pass unimpeded, or extends the path (e.g. by adding a loop of fibre cable) that delays the transit of the particle down that branch. When the particle traverses through the system, if both boxes are empty, or both boxes contain a loop, then the probability waves arrive in phase, and the particle goes to the upper detector with certainty. On the other hand, if only one of the boxes contain a loop, the probability waves arrive out of phase, and the particle goes to the lower detector with certainty. As such, by sending a single particle through the device, one can tell whether the contents of the two boxes are the same (i.e. fixed) or different (i.e. balanced).

This is one of the most fundamental quantum algorithms, and a key motivation of the field of quantum information science. In the context of a project in which quantum ideas are applied to music, this is hence a natural set-up to explore musically.

The Musical Interferometer

How can the above concepts of interference be expressed musically? The wave example involving two pure tones (discussed in the previous section) technically follows this form. Practically, however, this is nearly impossible to engineer except for very boring sounds (sine waves) in carefully controlled environments. Consider if two violinists in an orchestra played the same note at the same time. No matter where one stood in the audience, the sound of one violin would never cancel out the other. This is because there is too much variation between the instruments themselves, and reflections of the sound off the floor, walls and ceiling of the room all contribute to randomizing the relative phase with which the sound waves arrive. As such, one only ever hears *incoherent mixtures* of sounds in the context of classical music.

As part of a project to demonstrate quantum phenomena musically,4 we want to engineer the desired behaviour of superposition and interference. Using electronic instruments, we can simulate this behaviour at a musical level, rather than a sound level: that is, pertaining to which notes are played and when, rather than the sound waves emitted. Two keyboards can be configured such that if the same note is played on both keyboards at the same time, no sound is produced (see fig. 2). Each keyboard behaves like a branch of the Mach–Zehnder interferometer, where a sound is only produced if the particle travels into the lower detector. Holding down a key on a keyboard is equivalent to inserting a loop of fibre into the associated branch (fig. 3). A note only sounds if the key is pressed on one and only one keyboard. Equivalently, this is a musical version of Deutsch's algorithm, whereby f(0) = 0 or 1 depending on

4 The Quantum Music project, co-financed by the EACEA within the programme *Creative Europe* (559695-CREA-1-2015-1-RS-CULT-COOP1). Cf. "Editor's Foreword" in this volume (Medić 2018: 11-12).

whether the key on the first keyboard is up or down respectively, and f(1) = 0 or 1 depending on the state of the key on the second keyboard. The output signal follows from the comparison of the two functions f(0) and f(1).

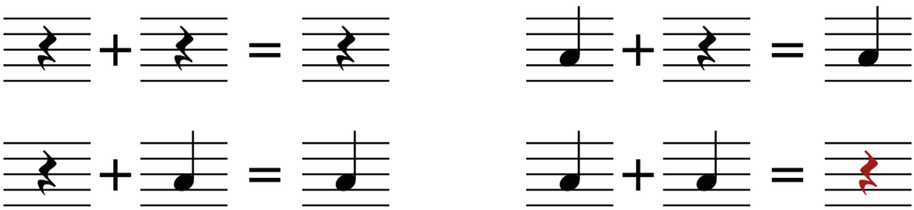

Figure 2. Musical interference. *Each equation indicates the state of the key (down or up) on the two input keyboards, and the resultant output signal. A note is sounded if one and only one of the keyboards is playing it. This is equivalent to a logical exclusive-OR (XOR) operation performed at the level of notes. Interfered keyboards differ from a pair of classical pianos in that if both pianists play the same note at the same time, no sound is produced by the musical interferometer.*

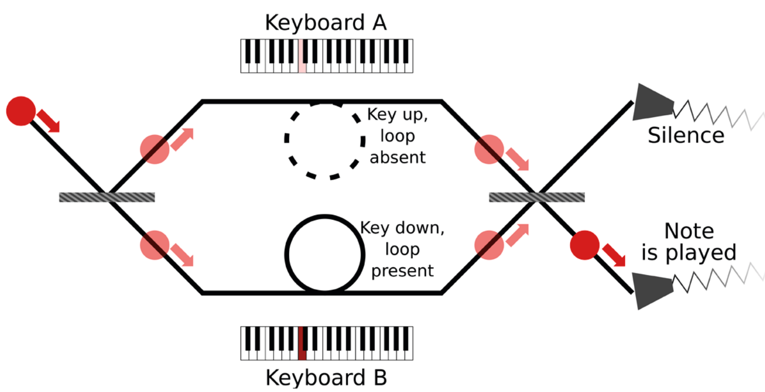

Figure 3: Musical Mach–Zehnder interferometer. *At each branch of the Mach–Zehnder interferometer, a loop of fibre is either inserted or not depending on the state of a key on a keyboard located at that branch: down or up. The loop's length is chosen such that inserting one loop will put the two branches perfectly out of phase, whereas if both loops are inserted the paths are once more in phase. The detectors are configured so that if the lower detector is triggered, a note sounds, whereas no sound is emitted when the upper detector fires. Hence, the upper detector is triggered when neither or both keys are pressed, and the lower detector when only one of the keys are pressed. The resultant musical behaviour of this device is detailed in Fig. 2.*

To extend this beyond a single note, one can configure each different musical key to independently interfere between the two keyboards (e.g. C3 on one keyboard interferes with C3 on the second, and likewise for every other pair of keys). Conceptually, this is like having a separate Mach–Zehnder interferometer for every musical key.

This opens up interesting compositional possibilities. Firstly, there will be *rhythmic interference* if the music for the two keyboards occasionally play notes at the same time. In this set-up, one rhythm could be played on one keyboard, and another on the second. When both are played at the same time, instead of hearing the two together, destructive interference results in a new tune with an entirely different rhythmic character (see example in Fig. 4). One interesting emergent phenomenon of this type of interference is that two input rhythms that are on beat become syncopated if the note length varies between the two keyboards (see Fig. 5).

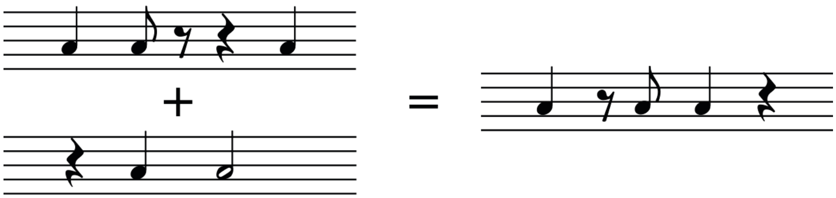

Figure 4: Rhythmic interference. *A note only sounds when only one pianist is playing it. Two different rhythms, when played together, produce a new third rhythm.*

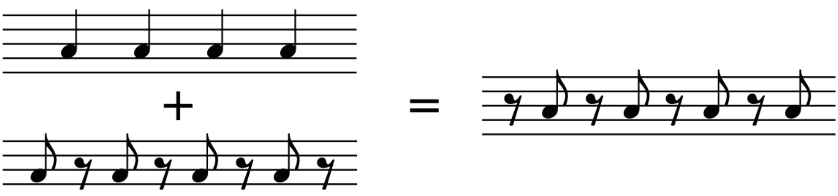

Figure 5: Emergent syncopation. *Both keyboards can be played on the beat, but due to differing note-lengths, the pattern output will be syncopated.*

There are also harmonic possibilities for interference, where two chords played on separate keyboards interfere to produce a new third chord. Suppose one keyboard plays C-major (C, E, G), and the other keyboard plays a C and B. The two C's destructively interfere such that the resultant chord actually sounded is E minor (E, G, B). (Further examples in Fig. 6.)

Figure 6: Harmonic interference. *Common notes (highlighted in red) between two chords are eliminated, and a different chord is heard than that played by either pianist in isolation.*

These two types (rhythmic and harmonic) of interference can be combined, such that a cadence may be split over two keyboards (whereby one keyboard plays the notes that resolve the chord played on the other), as drawn in Fig. 7.

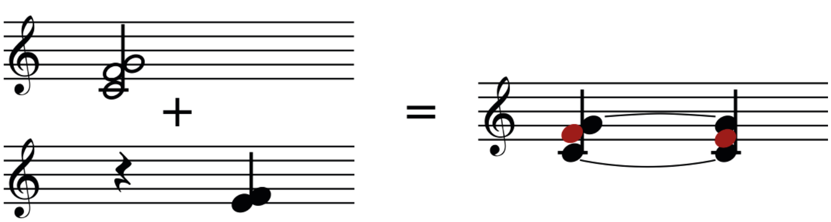

Figure 7: Distributed cadence. *(Notes that change in the output are highlighted in red). The cadence begun on one keyboard can be resolved through the notes played on the second.*

Finally, if the two keyboards are played together in perfect unison, then no sound whatsoever will be emitted from the interferometer.

Implementation

Schema

Having defined the effect of the musical Mach–Zehnder interferometer on notes, I now outline a mechanism by which it can be implemented with two piano keyboards. The most direct scheme to implement this instrument is drawn in Fig. 8.

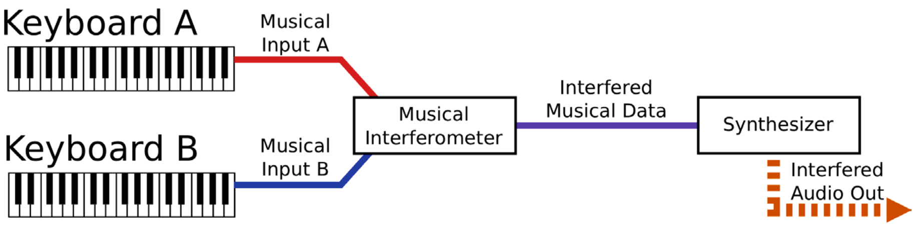

Figure 8. Instrument overview: direct scheme. *The "musical interferometer" detailed in this document works by manipulating musical data (e.g. a MIDI stream). Two keyboards send signals on separate input channels to the musical interferometer. This interferometer converts these input streams to a single channel "interfered" otuput stream. The interfered output stream is then input to an independent synthesizer to be finally rendered into sound.*

This proposal says nothing about the synthesization (notes to sound) aspect of the instrument. Rather, this instrument acts on the *musical* level, rather on the sound level, and sits in the signal chain in a similar position as an arpeggiator would: the unusual distinction here being that the interferometer takes in two inputs. The sound generation itself could be done by any synthesizer, e.g. software or hardware synthesizers, sampler, or possibly even exotic instruments from the Quantum Music project i.e. the hybrid piano (Novković et al. 2018).

The direct scheme of Fig. 8 is the simplest demonstration of interference. However, this scheme effectively collapses two instruments into one instrument with two controllers: once the notes have been processed by the interferometer, they cannot be assigned to as having originated from either keyboard. Moreover, from the very nature of the interference being simulated, there is no audible distinction between both keyboards playing the same note and no-one playing at all: as such, any calculated output could just as well have been played conventionally by a single player on a single keyboard.

As such in this direct scheme, the interfering behaviour is much more noticeable to those playing the keyboard (since the output is different from normal playing expectations) than it would be to the audience who only listen to the output. This makes this direct scheme is trickier to use in the context of a piano duo concert – though it could be combined with clever visual aids (most straightforwardly making the pianists' hands on their keyboards visible to the audience) to convey the concept of interference. On the other hand, this scheme is ideal for an installation piece, where both keyboards are available for audience interaction. Alternatively, this could be combined with a piano soloist playing one input, and a limited audience wherein each audience member controls a separate second input – every member of the audience would then hear a different output (e.g. through headphones) resulting from their interference with the pianist.

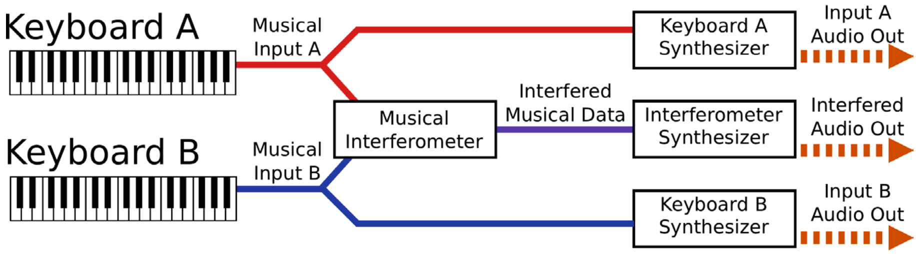

Figure 9. Instrument overview: hybrid scheme. *In addition to the interfered output, in the hybrid scheme the two input channels are also directly rendered into sound, so that the audience can hear both the inputs and the interfered signal together.*

An alternative *hybrid scheme* (see Fig. 9) is more suited for performance by piano duo. In this scheme, the input signals are *also* rendered into sound, alongside the interfered output. Here, the audience will hear three distinct instruments: two directly controlled by the performers, and one emergent from the interference between them. The sonic aspects of the three synthesizers can be chosen independently (for example, the two input signals could be rendered as grand piano sounds panned to left and right, while the interfered signal could be sent to a more obviously electronic synthesizer). This scheme can also be implemented semi-acoustically, through use of *hybrid pianos* – concert pianos which emit both MIDI data as well as functioning as acoustic instruments (Lončar and Pavlović 2018). Here, there is no need to explicitly render the two input signals into a sound using synthesizers, since this aspect of the music will have already been made audible by the piano's acoustic mechanism.

Implementation by MIDI manipulation

Finally, let us consider some more specific remarks on how the musical interferometer itself could be created using commonly–available musical technology, namely, Musical Instrument Digital Interface (MIDI).[5] Here, one requires two MIDI-controller keyboards[6] to send MIDI data on separate channels (e.g. channels 1 and 2) into a single computer that runs the *musical interferometer* program. The musical

5 MIDI Manufacturers Association Incorporated, "Summary of MIDI Messages," Table 1, http://www.midi.org/techspecs/midimessages.php

6 Equivalently, one could use the electronic output of the hybrid-piano mechanism (Cf. Novković et al. 2018).

interferometer program outputs a new MIDI stream, satisfying the rules outlined in the previous section. This interferometer program could be implemented as a Virtual Studio Technology (VST) plugin within a digital-audio workstation (DAW), or as a script in a DAW-specific programming language.[7] The output of this plugin is will then also be MIDI data on a new channel (replacing the inputs channels in the direct scheme), which can either be fed directly into a software synth on the same computer, or send as MIDI out from the computer to be rendered into sound elsewhere (e.g. to a hardware synthesizer).

MIDI controllers send a digital signal whenever a key is pressed (**ON**) or released (**OFF**). This signal encodes information about which key is pressed, and the velocity with which the key was struck. There is no continuous broadcast of which keys are held down, which is why lost signals can sometimes result in "sticky notes" that do not end even when all keys on the keyboard are up.

To implement the musical interferometer, the following behaviour is required. The musical interferometer takes MIDI signals as its input, and must remember which keys are being held down on each keyboard. An MIDI signal is output from the interferometer whenever an input signal is received. When a **ON** is received from one keyboard, if the note is not held down on the other keyboard, then **ON** is output from the interferometer. If the note is down on the other keyboard, then **OFF** is output instead. If two **ON** signals for a note are received within the same time window, then no signal is transmitted. If **OFF** is received from one keyboard and if the note is held down on the other keyboard, then **ON** is output. On the other hand, if the note is not held down on the other keyboard, then **OFF** is sent. If two note release signals are sent within the same time window, then **OFF** is transmitted.[8]

Finally, some timing considerations need considering. Digital audio typically renders at 44,100 or 96,000 samples per second, and synthesizers handle inputs such as key presses or releases every 100–200 samples. This means the the length of time the system takes to respond to an input is extremely short (order of 1 mS) compared to human reaction times. In the context of this musical interferometer, this may result in unwanted notes being output for a very short period of time equal to the actual difference in time between when the two pianists press the same key – even if these notes would be considered pressed "at the same time" by usual musical standards. (A similar problem will occur for "simultaneous" releases). If the synthesizer rendering the output has a slow attack time (e.g. is a "synth pad" or simulated string instrument), this may not be a problem, since these ghost notes will not reach significant volume before they are released. On the other hand, if a more percussive sound is used, such as that of a piano, then this could cause unwanted audio problems.

7 One might also consider a dedicated hardware solution by preparing appropriate microcontrollers, or using a small programmable device such as a Raspberry Pi.

8 Technically, no signal is required if two release signals are detected simultaneously, since the state prior to release would have been that both keys are down, which should also have resulted in an **OFF** output signal. However, since most synthesizers function correctly even if multiple **OFF** signals are sent, this redundant behaviour is suggested to minimize the chance of sticky notes.

One way to smooth over this is to introduce temporal quantization – dividing the compu- tation into discrete time steps covering a longer period of time, such that all signals received within this window are treated as if they arrived simultaneously. Such quantization can be introduced in the form of a delay buffer: when one input signal is received, a degree of time (adjustable logarithmically around 0.1–100 milliseconds) must pass before it is acted on – and if the second input signal is received within this window, then the output is adjusted accordingly. The shorter this time window, the more precisely synchronized the pianists must play in order for their music to interfere correctly. However, if the time window is too long, this will introduce noticeable latency into the signal chain, which the performers may find musically distracting (leading to a sense of disconnection between the notes they play and the sounds they hear). It is thus advised that this value is tuned to the lowest value that the players can successfully perform with.

Conclusion

In this article, I have presented a design for a new musical device: the musical Mach–Zehnder interferometer. Unlike a typical instrument, where two instruments playing at the same time add their sound together, here the two keyboards have the potential to cancel each other out. This opens up emergent possibilities for composition, wherein two players generate three sounds, the third as a strict function of the two that are directly input. Although straightforward in concept (no more complicated than a logical "exclusive or" gate), I envision such a device could play a role in the context of a performance themed around quantum theory, since it demonstrates one of quantum theory's core concepts: interference.

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Ендру Гарнер

Музички Мах-Цендеров интерферометар

(Сажетак)

Феномен интерференције налази се у средишту квантне физике и одговоран је за многе необичне аспекте понашања квантних честица, који одступају од наших свакодневних очекивања. Премда класична физика дозвољава интерференцију таласа (нпр. звучних таласа), у квантној теорији могуће је да се ефекти интерференције одразе и на појединачне честице. Уређај који ово експериментално демонстрира јесте Мах-Цендеров интерферометар: једна честица (нпр. фотон) путује једном од две могуће путање, а квантна интерференција између ове две путање афектује финалну позицију на коју честица стиже. У овом чланку предлажем механизам којим се може музичким путем демонстрирати квантна интерференција појединачних честица: музички Мах-Цендеров интерферометар. Овај нови квантни инструмент користи две међусобно независне електронске клавијатуре, чији су аутпути у интерференцији према правилима Мах-Цендеровог интерферометра. У чланку разматрам музичке могућности које овај инструмент нуди и скицирам како га је могуће конструисати путем софтверске симулације. За разлику од конвенционалних инструмената и околности да, када два инструмента симултано свирају, њихови звуци се спајају и међусобно појачавају, овде две клавијатуре имају могућност да једна другу – укину. Ово отвара неслућене могућности за компоновање, јер два извођача могу генерисати три групе звука, где трећа настаје стриктно као функција претходне две, које представљају директне инпуте. Мада је замисао оваквог инструмента сама по себи прилично једноставна, указујем да овакав уређај може да има специфичну улогу у оквиру музичког извођења инспирисаног квантном теоријом, јер демонстрира један од њених кључних концепата: интерференцију.

Кључне речи: квантна физика, интерференција, Мах-Цендеров интерферометар, нови музички инструмент